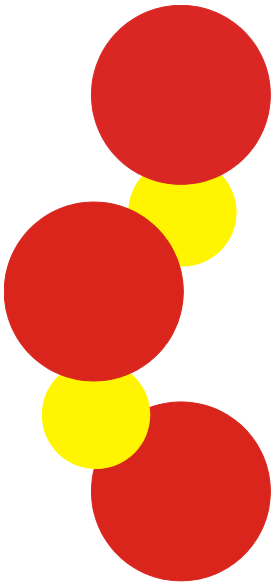
Working Mathematically with Infants

Enhancing Number Sense by Engineering 'aha' Moments

Doug Williams



Working Mathematically with Infants

Part 1 Only
Distributed for review

Years K, 1 & 2

Poly Plug
Calculators
Threaded activities
Investigations
Detailed curriculum planning

Doug Williams

Derived from Calculating Changes, enriched by the Task Centre and Maths300 and integrated with Maths With Attitude. ***Working Mathematically with Infants*** supports teachers to construct curriculum around learning to work like a mathematician.

First published 2010. Reviewed 2014. Reviewed 2015

ISBN: 978 0 9807630 0 3

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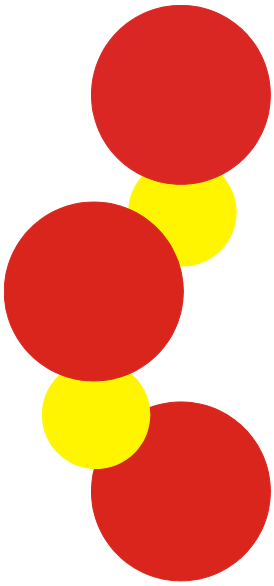
Part 1: Preparing To Teach

Our Objectives



- To engineer the classroom so that 'aha' moments in number happen more often for more children.
- To develop children's number sense beyond what is normally expected for their age.

Our Attitude...

- ...to learning:

  learning is a personal journey stimulated by achievable challenge

- ...to learners:

  stimulated children are creative and love to learn

- ...to pedagogy:

  the art of choosing teaching strategies to fascinate, captivate and absorb all children

- ...to mathematics:

  mathematics is concrete, visual and makes sense

- ...to learning mathematics:

  all children can learn to work like a mathematician

- ...to teachers:

  the teacher is the most important resource in education

- ...to professional development:

  teachers improve their teaching by re-enacting stories of success from the classrooms of their colleagues

Working Mathematically with Infants *is constructed from stories of success collected and re-enacted over years through the Calculating Changes network.*

It offers ten weeks per semester, in each semester from Year K to Year 2, of Threaded Activities and Investigations. To support the implementation of these learning experiences the kit includes a class set of Poly Plug and assumes children have free access to simple four function calculators.

Membership of Calculating Changes is a prerequisite for WMI as the full, and latest, information about any Threaded Activity is found on the web.

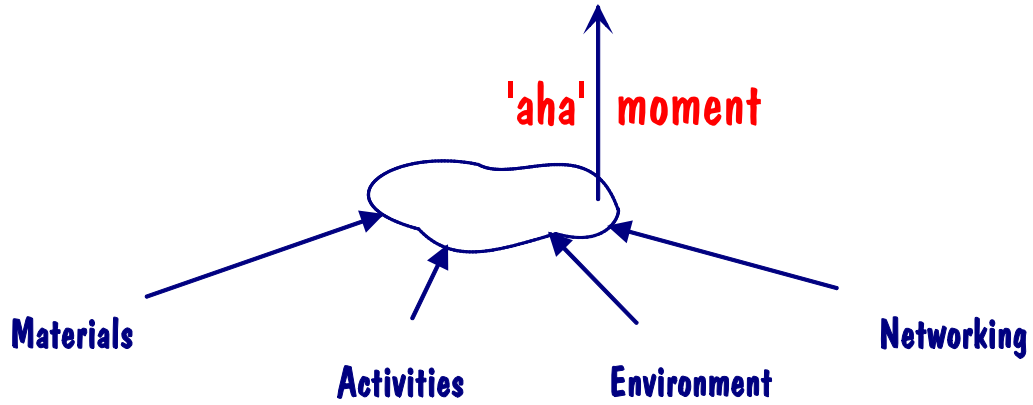
Engineering 'aha' Moments in Number

The Calculating Changes network was established in 1996 to draw together earlier work on the use of calculators, concrete materials and constructivist theory in a web-based community of teachers.

Two major influences on the direction of the network were the:

- Calculator Aware Number project in Britain in the 1980s and early 90s
- growing recognition of the educational value of Poly Plug, a manipulative invented in 1993/4

Over time network teachers grew to see themselves as attempting to engineer best practice classroom environments to encourage children to create their own learning. Through sharing ideas and experiences across the network, teachers and children alike began experiencing more and more 'aha' moments.



Through classroom trial and reflection teachers begin to alter their vision of what children can do. Deeper and richer activities, and more open-ended teaching practices are introduced. The outcome is children's number sense is enhanced.

Drawing from these years of development ***Working Mathematically with Infants*** chooses to build an environment which...

- encourages open access to materials
- encourages open access to calculators
- creates time for children to construct their own learning
- builds on learner's efforts
- values learning in community
- encourages mathematical conversation
- celebrates 'aha' moments
- applauds mental strategies
- asks *Can I check this another way?*
- challenges current conceptions
- supports risk-taking

and use activities which...

- are colourful & tactile
- encourage learner ownership
- involve personal recording in a maths journal
- offer a partnership between concrete, symbolic and personal recording

- encourage mathematical conversation
- are non-threatening
- uplift the learner (they feel better about themselves)
- place number sense in problem solving situations
- offer opportunity to revisit and be challenged anew and therefore can be threaded

Problem solving situations are included in ***Working Mathematically with Infants*** as whole class Investigations which model learning to work like a mathematician.

Threading

- Rich tasks
- Familiar structure
- Fresh challenge
- Short, frequent visits

Threading is a teaching technique requiring rich activities used 2 or 3 times a week for a few minutes each time over several weeks. The structure of the activity remains constant (and therefore familiar), but the challenge within it is fresh each time.

The activity appears as a thread in the fabric of the curriculum, as in the Planners (pp. 16-21).

Threaded Activities give children time to construct their own learning, eases preparation because teachers do not need to constantly look for 'something new in maths' and offers opportunity to gather assessment information as you share time with groups.

> *It was like a journey that we all approached together because we were all students; it became extremely enjoyable and a very valuable learning experience. The children were keen to do these activities and even the repetition made them feel more secure and none of them complained.*
> Kate Thureau, Poly Plug, Proportion & Percent, member activity for Year 3+

Teacher Comments

The following testimonies were offered by infant teachers in their evaluation of Calculating Changes professional development courses. These teachers attended between two and five linked courses with classroom trialing sandwiched between. Comments are taken from the Evaluations section of the Calculating Changes web site.

Teacher A ... Prep

- *Hands on fun, children confident in working through activities, chance to use mathematical language. Easily linked with other subjects - takes fear out of maths.*
- *Poly Plug is a great resource for use in most areas of maths. Can be used in both language and maths. Used almost daily.*
- *Maths is now fun - I've been able to extend and adapt activities and ideas to suit children's levels and stages of maths.*

Teacher B ... Prep

- *Makes maths more exciting and enjoyable.*
- *Makes children more questioning of the whys and hows of what they're doing.*
- *Children being able to explain the processes involved in solving problems*
- *A more hands on approach to maths.*
- *Poly Plugs - a fun aid for calculation.*
- *Leading children to more 'aha' moments.*

Teacher C ... Grade 1/2

- *Wonderful, practical ideas. Very hands on. Helps children to fully understand maths not just calculating without understanding.*
- *Poly plugs - activities are endless, really supports children.*
- *Children are often heard to say I'll just get a calculator for that.*
- *Builds my pedagogical skills, thus the children are more likely to learn!*

Teacher D ... Grade 2/3

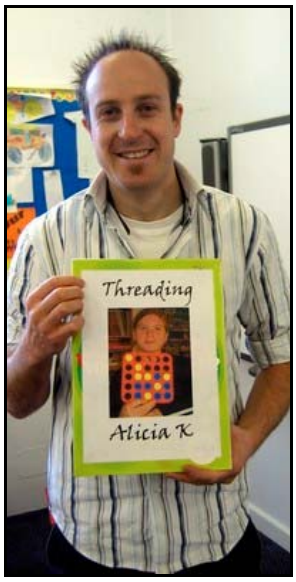
- *Ideas and activities to connect kids to be interested in maths and think/articulate for themselves. Involve parents wherever possible. Promote correct understandings.*
- *Poly Plug is a quiet, attractive concrete aid to enhance understanding. Great for games.*
- *Wonderful to see how children enjoy their learning.*

Threading Works

This report is extracted from *Reflections on Engineering 'aha' Moments in Number* by Nicholas Dale which is stored on the Maths Task Centre site. Nick was working as a Year 3/4 teacher at the time and attended this course with a representative from each level of the school. His report describes changes in his classroom and across the school.

Part of my current leadership role is to deliver professional learning to my colleagues. I have presented a number of sessions from this initiative and all teachers have taken it on. The junior primary teacher has found this to be one of the best teaching tools and uses aspects of it in her programming everyday. I have modelled lessons to staff, organised staff to visit other teachers at other sites to extend learning within their year levels, and had them share their experiences at hub meetings within the district.

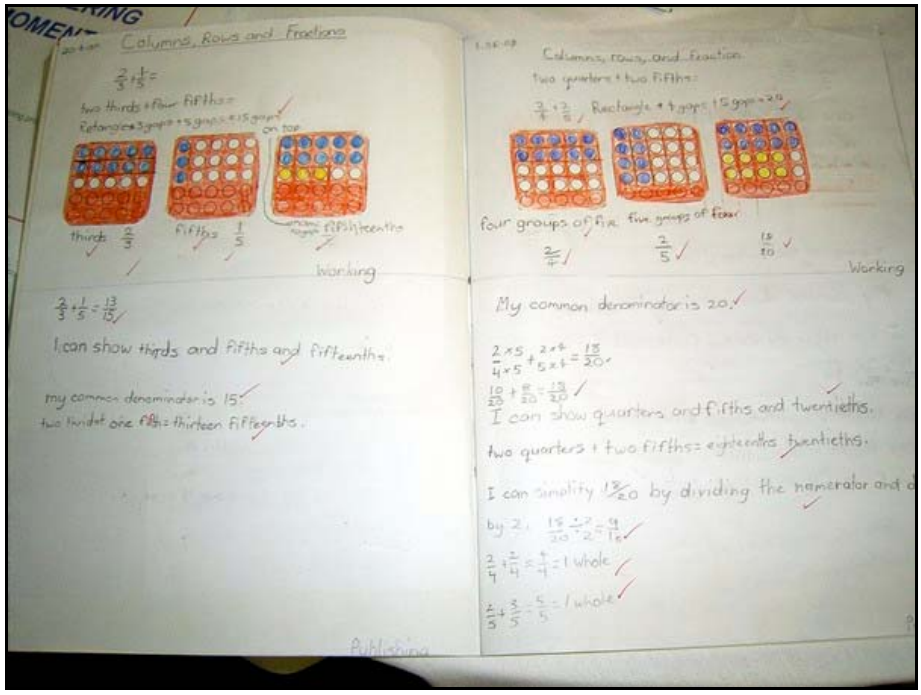
I have developed a school wide mathematics proforma that incorporates Threading activities from Calculating Changes. This ensures that teachers continue to plan these rich mathematical programs each day.



Students have been more focused, engaged and interested in mathematics. They now enjoy doing maths everyday and look forward to the tasks I present to them. (Our curriculum...) has moved away from the textbook oriented lessons and into creativity in maths. Lessons in which I used to struggle to engage students for 30 minutes, now engage them for sometimes up to two hours. Therefore, this idea we have been presented with WORKS!

The Threading scrapbooks have been an excellent way of recording the students' work. It is a way for me to check levels of understanding, and for students to refer back when we are threading an activity for a number of weeks. Rather than asking what to do next they have a valuable tool for checking themselves. It has created students who have become more independent and take on more responsibility for their own learning. Also I have been able to use their books as a way of presenting ideas to other teachers.

I believe recording is an important component not only for assessing and education department requirements, but for the students as well.



During this year I have attended a number of mathematical professional development sessions. *Engineering 'aha' Moments in Numbe*r (see Professional Development Purpose p. 12) has been the most informative and practical and has shown an improvement in learning outcomes by far. I was at a point where I was not enjoying teaching maths and the students were not enjoying learning about it as well. I was looking at developing different approaches to delivering a more interesting maths program. During this program I have grown in myself and introduced a new methodology that has been successful.

When reporting to parents I talk about the student's learning in using this program and how these skills transfer. I use the Threading Book as evidence of what skills I am covering within my units and this is a good evidence based document.

Resources

Your kit includes Calculating Changes site membership for the whole school, 25 Poly Plug and this manual. Threaded Activities for infants are outlined in the manual, and described in full on site. Companion investigations are explained in more detail in the manual and in most cases are reference to further information at Mathematics Centre or Maths300.

Planners in the manual (see Pages 16-21) detail 10 weeks of integrated number work for *each semester* in *each year* from K - 2. Threaded Activities and Investigations work together throughout the program to enhance number sense within the umbrella context of all children learning to work like a mathematician.

To achieve this objective we also expect you will have simple four function hand-held calculators in your school. In addition some Investigations need other materials generally found in schools. See the list below.

It is *not necessary* for your campus to belong to Maths300 to make full use of ***Working Mathematically with Infants***, but those who do will find additional support, especially in the form of software to extend some Investigations.

However, if your school does belong to Maths300, then it may also build the curriculum for Year 3^+ around the Maths With Attitude kits. (See p. 90 for further explanation of these kits.) ***Working Mathematically with Infants*** is designed to integrate conceptual development, content knowledge, teaching craft and documentation with Maths With Attitude. Schools that choose this suite of resources are able to build curriculum around a unified approach throughout primary school. This approach, summarised by our attitudes on Page 2, is consistent and developmental in language, teaching craft and mathematical content. All content is reflected in the requirements of any official curriculum documents of any system.

Poly Plug

Threaded Activities assume open access to Poly Plug because nothing else offers so much in the one resource. Besides, children simply love using them. Further, where appropriate, Poly Plug is used in the Investigations, so you gain maximum benefit from the resource.

Poly Plug...

- is colourful, tactile, motivating
- is classroom savvy - noiseless, easy to pack up
- is useful as both a structured and an unstructured aid
- has applications across a broad range of mathematical content - pattern, counting, fractions, graphing, symmetry, problem solving
- has even more applications when used in conjunction with other simple, readily obtainable material, such as dice

Calculators

Calculating Changes assumes children have free access to calculators from the day they enter school. This assumption is based on research from the Calculator Aware Number (CAN) project in England, and its derivatives elsewhere.

Given this assumption we want children to use calculators that give correct answers, and the majority of four function calculators in schools don't!

To test your calculator enter this equation in the order shown: 2 + 3 x 5 =

The correct answer is 17. If your calculator gives the answer 25 it has not been programmed to interpret the order of arithmetic operations. That is, it does not have an Algebraic Operating System (AOS). If you are only using matched pairs of operations +/- or x/÷ there is no problem. However, if you are allowing free access to the machines, as assumed in Calculating Changes, children will press combinations of operations at random. Would you prefer that the calculator then gives the correct answer?

If the activity involves the possibility of mixing the operations of + or - with x or ÷, most simple four function machines will give the wrong answer to most questions. So, use your school calculators if you wish but beware of this limitation.

For further information visit the Materials link at Calculating Changes which lists the brands we currently know that are simple and operate algebraically. We are not suggesting you rush out and purchase these. We just want you to be aware of the limitations of what you may have and be aware of possibilities.

http://mathematicscentre.com/calchange/cchpp_mm.htm#calculators

Calculator Aware Number Project

The report of the Calculator Aware Number (C.A.N) project was published in 1991 under the title *Calculators, Children and Mathematics* by Simon & Schuster on behalf of the National Curriculum Council, England. Calculating Changes has been granted permission to use any part of the CAN Report (which is now out of print) to achieve its aims. The following is taken from Page 12 of the report.

> From the beginning of the project, children were allowed free access to calculators alongside other apparatus. A report from one LEA (Local Education Authority), after three years work, describes the significance of this:
>
> A significant finding from the project is that calculators should be viewed as an item of multi-purpose mathematical apparatus and that teachers do not need to design specific tasks to bring them into play. Pupils select the calculator as they feel it is appropriate to the task in hand just as they would select other material - Multilink, counters, modelling apparatus etc. to solve the problem they are dealing with. [Durham County Council 1989]
>
> However, the frequent use of calculators has made the mathematical experience of project children different from the experience of other children. Many children have developed mathematical concepts and methods which have not in the past been expected at their age. For young children, the calculator is a toy, but it is the first toy for young children that incorporates the number system. As children play with their calculators, they find out a great deal about how numbers behave.
>
> Ready access to calculators has also given the children great confidence. They are never faced with calculations that they cannot do. Their problem

> now is to decide the appropriate calculation to do in order to solve a problem, and to interpret the results of that calculation in the problem situation. Exploration and investigation have taken over from the repetitive practice of calculation as the usual style of number work.
>
> …
>
> Most children in the project have also decided for themselves that they do not need, or want, to be dependent on their calculators for all calculation. It is often faster and easier to do a calculation mentally, and children sometimes vie with one another to extend their skill in mental calculation. The project has seen a great flowering of mental calculation, often led by the children.

Funded by the British national government, C.A.N began in 1985 with about 20 classes of children aged six in 15 schools. Over the years the number of children involved grew from the hundreds into the thousands as more schools joined the project. Significantly, all the originating schools and Local Education Authorities stayed with the project throughout its lifetime. The project was led by Hilary Shuard, Homerton College, Cambridge University.

The C.A.N. report is liberally sprinkled with quotes from teachers, head teachers and others who evaluated the project. For example, on Page 9 one Head Teacher comments:

> *The spreading of the CAN philosophy in school has been rapidly accelerated by the movement of an experienced teacher who was involved in the first year of CAN with a class of six and seven-year-olds, to a reception class of four and five-year-olds. This teacher has used the methods, and many of the activities and ideas that she developed with her six and seven-year-olds, with great success. This approach has spread sideways, as the two other reception teachers have seen how the children have developed mathematically.*

Use your Calculating Changes membership to find more information from CAN in both the Free Tour and Members sections.

Working Like A Mathematician

Our attitude is:

all children can learn to work like a mathematician

What does a mathematician do and why do they do it?

The Mathematics Task Centre Project has probed this question with several professional mathematicians in an attempt to identify the process for classroom use K through 12. The result is the description on the next page. In particular we are indebted to the clarity provided by Dr. Derek Holton on this matter.

Perhaps the most important aspect of Working Mathematically is the recognition that *knowledge is created by a community and becomes part of the fabric of that community*.

Recognising, and engaging in, the process by which that knowledge is generated can help children to see themselves as able to work like a mathematician.

Hence learning to work like a mathematician is the scaffolding of ***Working Mathematically with Infants***. The Working Mathematically Process is described on the next page in a language which can be used with children from the day they enter school.

It is probably worth noting that one Year 1 child asked what mathematicians do replied, with her face full of wonder:

> *They solve the world's hardest problems.*

By comparison, many children in later years would reply:

> *Sums*

Skills, Strategies & Working Mathematically

The Working Mathematically description places learning mathematical skills and problem solving strategies in their true context.

Lessons on skills or lessons on strategies are not an end in themselves. They provide the toolboxes that mathematicians carry in their struggle to solve problems.

- **Our skill toolbox** can be added to in the same way as the mechanic or carpenter adds tools to their toolbox. But equally, the addition of the tools is not for the sake of collecting them, but rather for the purpose of getting on with a job. A mathematician's job is to solve problems, not to collect tools that might one day help solve a problem.

- **Our strategy toolbox** has been provided by the joint wisdom of the mathematicians of the past. All mathematical problems (and indeed life problems) that have ever been solved have been solved by the application of this limited set of strategies.

Working Mathematically

First give me an interesting problem.

When mathematicians become interested in a problem they:

- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:

- Can I check this another way?
- What happens if ...?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:

- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:

- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- Break a problem into smaller parts
- ...

If one way doesn't work, I just start again another way.

Professional Development Purpose

Our attitude is:

the teacher is the most important resource in education

> I now perceive the project as:
> *... very worthwhile. Takes patience with Preps, but allows the children to leap ahead at their own pace.*
> The response of my children has been:
> *... enthusiastic. They use them* (Poly Plug) *whenever they want to help them with all sorts of tasks.*
> The continuing steps for myself are:
> *... trying new things; refining them; writing them up; sharing them.*
> The continuing steps for the school are:
> *... recording what we do; sharing what we do; keeping new staff informed; keeping the momentum going.*

This teacher's evaluation is one of many from years of Calculating Changes workshops. Their greatest success comes when they are held as linked experiences over time with classroom trials sandwiched between. The six day Maths on the Move program *Engineering 'aha' Moments in Number K-8* has proven particularly successful for primary schools. Nick Dale's report on Page 5 is an example of the shifts in personal teaching practice, children's learning and school wide curriculum development that result from this course. One day courses and after school sessions are also available.

Research suggests that the professional development most likely to succeed:

- is requested by the teachers
- takes place as close to the teacher's own working environment as possible
- takes place over an extended period of time
- provides opportunities for reflection and feedback
- enables participants to feel a substantial degree of ownership
- involves conscious commitment by the teacher
- involves groups of teachers rather than individuals from a school
- increases the participant's mathematical knowledge in some way
- uses the services of a consultant and/or critical friend

Working Mathematically with Infants has been designed with these principles in mind. All the materials have been tried, tested and modified by teachers from a wide range of classrooms. If we can't be present to lead your professional development sessions, then we hope ***WMI*** will enable teacher groups to lead themselves further along the professional development road, and will support systems to improve learning outcomes for children K-2.

For more professional development information see Page 86 and, for external assistance with professional development, contact:

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Part 2: Planning Curriculum

Curriculum Planners

Our attitude is:

learning is a personal journey stimulated by achievable challenge

Curriculum Planners on the following pages integrate the use of **Threaded Activities** and *Investigations* into 10 weeks of concrete, visual, open-ended, challenging number work per semester, for each semester in each year from K to 2.

- *Typically Threaded Activities are used for a few minutes each day, several days per week over several weeks.*
- *Typically Investigations are used for a block of time - often much more time than is usually set aside for a maths period.*

Planners provide a starting point for those new to these materials and a flexible structure for those more experienced. You will need to map the Planner weeks into your local curriculum document. Different teachers and schools will timetable the same planners in different ways, but all will be making use of the same extensively trialed materials, activities, investigations and pedagogy, which for others has produced:

happy, healthy, cheerful, productive, inspiring classrooms

Threaded Activities are shown in bold.

- The first time one is used it sometimes takes a substantial part of a session to introduce. Therefore they are shown as the only activity during this introduction session. You will also need other 'standby' activities for these sessions.
- Introduction sessions are highlighted with a background colour.
- Identifying the first use of a Threaded Activity in this way also serves to identify the introduction of new content in the development of the planning sequence.
- Threaded Activities can be used with the whole class as starter activities; and can be used as small group activities (including with your teaching group); and can be combined with any other best practice you already use. Their critical property is that, once learnt, they need to be used for 10-15 minutes per day, several days per week over several weeks. Planners have been structured in this way.

Investigations are shown in italic.

- Investigations frequently captivate children and need to be continued beyond a single time slot. Therefore they are shown in the first session of a week in the expectation that they will need to run over more sessions.
- Sometimes teachers continue an Investigation into the following time slots of the same day. Sometimes continuation is built in as group work in mathematics slots on following days.
- Investigations will teach, review and develop content but their main purpose is to absorb children in learning to work like a mathematician (see p. 11).

Planning Points

- Planners assume four lessons per week of about 1 hour each.
- Planners *do not* prescribe a continuous block of work. Use them flexibly.
- Planners offer *only* 10 weeks per semester and those weeks are almost exclusively in number. They do not represent a full year's curriculum.
- Map these weeks into your program then complete your year as appropriate.
- There *is* a developmental flow when the planners are viewed as a whole from Year K to Year 2, however, there is plenty of room for flexibility within and across Year levels. Your children will guide you.

Using The Planners

- Choose a week. It is most likely to include both Threaded Activities and an Investigation.
- Find out more about the Threaded Activities by:
  (a) reading its introduction in the Threaded Activities section from Page 22.
  (b) exploring its depth on site.
- Find out more about the Investigation by reading its description in the Investigations section from Page 37.
- Descriptions are all the detail you need to explore the problem. However, most Investigations can be enriched by reading the additional detail stored in the web links listed for each investigation. Task Cameo links are public access. Maths300 links require membership of that site. These are the only additional links used.
- Map the week into your syllabus.

Questions

- Can I use Planner activities in weeks other than those ten?
  Absolutely. Planners offer a starting point. There will be many times when you or your children will want to continue using what the Planners offer.
- Can I use in my current year an activity or investigation listed for another year?
  Certainly, provided you break from the planning sequence as part of a team planning approach from Years K - 2.

Expect to be surprised by what your children can do!

Additional Resources

For the most part Investigations use Poly Plug or calculators or other materials normally available in classrooms. However you will also need:

Dominoes - Year 1 - *Domino Trails*
Cuisenaire Rods - Year 2 - *Rod Mats*

Further, many Investigations require printing and preparing game boards or other resources. Plan well ahead to allow enough time for this preparation.

In many Investigations, you will notice reference to the children's mathematics journal. In the early years this will be made for them page by page as they record their understanding. By Year 2 however, a self-maintained journal is an expectation. As well as consolidating learning, journals offer significant assessment information.