Maths *Not* At The Movies

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Dr. Burkard Polster and Dr. Marty Ross presented the closing address to the 2005 December Conference of the Mathematical Association of Victoria (MAV) under the title **Maths at the Movies**. *One problem in their address 'wouldn't let me sleep'. This 'letter' to the pair, written to address my insomnia, was first published in Vinculum, MAV, Volume 43, Number 2, June 2006.*

Dear Burkard & Marty,

Thanks a lot (*Not!!*) for breaking into my semi-conscious state on the morning of Saturday December 3rd 2005 with your stupid maths problem from the movies. You know the one - you showed it in your closing address at the conference the day before: *If I can paint a house in three hours and you can paint a house in five hours how long will it take if we do it together*. It was from that film where the kid somehow owned a baseball team and before the grown-ups could play ball they had to help him do his homework question.

Of course it was a stupid problem. How else was I going to get it out of my mind and drift into the sleep I deserved after a week of preparing for and presenting seven sessions at the conference? Everything was in my favour for Saturday slumber; the fluttering of the rain-showers on the tin roof of my Phillip Island retreat told me I needn't hurry into the garden; even the absence of birdsong was testimony to the intent of all living creatures to snuggle into the nest. Seven-thirty was too early for everyone on this particular drizzly morning. But that stupid problem!

Of course it was a stupid problem. It has no practical application. Even a painting contractor employing several workers isn't going to 'do the math' to work out to the fraction of a minute how long it will take two workers working at unequal rates to paint a house. A bit of estimation will do... (thinks, like in the comics)

> *If they work for three hours it's obvious that there will be one house painted by one bloke and three fifths of a house painted by the other. A lot of use that is when there is only one house to paint, so it must take less than three hours. Try two. Then one bloke has painted two thirds of the house and the other has painted two fifths. That's the same as ten fifteenths and six fifteenths, which is only just a bit over one house, so two hours is near enough.*

Okay you two. Because one of you can paint a house in three hours and one of you can paint the same house in five hours, I'm expecting you to work together on this one and I'll give you two hours to get it done.

Must have been a doll's house!

Be quiet you stupid problem I'm trying to drift off.

Yeah well suppose the new industrial relations laws were gazetted. Then the contractor would probably have given them only one hour and fifteen minutes to do the job and taken away their penalty rates, leave loading, WorkCare cover, Medicare benefit, Smoko and sacked them into the bargain if it wasn't done in time; unless of course they voluntarily came back to work in their own time - preferably Christmas Day - to ensure that the job came in before time and under budget!

All right! I give up! I'll think out the answer to your stupid problem and then maybe you will go away. But I'm not getting out of bed to get a piece of paper. (Thinks ...)

> *With any luck this will be like counting sheep and I won't need to anyway. In the past, maths like this seems to have had an appropriate dozy effect on the not-so-helpful adults in the baseball team, and I have corroborative evidence of the same phenomenon from too many of my own classes.*

So, picking up where the contractor's thinking left off, we are looking for a number of hours x (that's original why don't you use h - go away!), which is less than two. In that time I will have painted x thirds of the house and you would have painted x fifths and together, one whole house would have been painted. So:

$$^{x}/_{3} + {}^{x}/_{5} = 1$$

$$^{5x}/_{3 \times 5} + {}^{3x}/_{3 \times 5} = 1$$

$$^{x(5 + 3)}/_{3 \times 5} = 1$$

$$\text{So, } x = {}^{3 \times 5}/_{(5 + 3)}$$

There, done!

But how do you know you are right?

Well x is just less than two - fifteen eighths, that's one whole hour and seven eighths more, just less than two! And besides, the smart guy in the movie clip said the answer was of the form $^{a \times b}/_{(b + a)}$ and Marty said that was correct, so my answer will do.

Oh come on, since when did you take the word of a two-bit actor who probably never played in another film, let alone the word of a mathematical juggler?

Go away! The pillow is over my head. I can't hear you!

Yes you caaan, ... 'cos I'm in your head under the pillow. Come on stand up and be a mathematician. You know the question. *Can I check this another way?*

Aaargh! All right, but I'm *only* rolling over. I'm not standing up. What's the problem again?

> *If I can paint a house in three hours and you can paint a house in five hours, how long will it take if we work together?*

Stupid problem! All right, our achievement ratio is 3 : 5. Right?

That's what the problem says.

Sure, but that is only when we think of the time involved. What about our achievement ratio if we think of the amount of the house painted.

Well you work faster than me...

...therefore, I will paint more of the house in the same time, so when we think of the amount of house painted, our achievement ratio is 5 : 3 in my favour. Five parts of the house will be mine and three parts will be yours and that's eight parts altogether.

Yeah but you are supposed to answer as time, not as parts of the house!

I know, but that means I will be using five eighths of the time I would take by myself, so five eighths of three hours, which is fifteen eighths of an hour, or one whole hour and seven eighths just like before ... and speaking of before, before *you* say anything, it also means that you have worked three eighths of your usual five hours, which is still fifteen eighths. So there! Now I'm going back to sleep.

No youur nooot... because now it's eight forty-five and you have to go to the shop to get some milk for breakfast. And before *you* ask, those numbers are kinda symmetric aren't they?

Wadda ya' mean?

Five eighths of three = three eighths of five. Suppose the ratio was 2 : 7. Would it be that seven ninths of 2 would be the same as two ninths of seven?

Of course. Both expressions are fourteen ninths. And before *you* ask, it will work like that for any ratio a : b

$$^{b}/_{(a+b)} \times a = {}^{a}/_{(a+b)} \times b$$

Well, unless you're a lazy sod, like I *haven't* been this morning thanks to Burkard and Marty, and your achievement ratio is zero. Then we can achieve nothing *together* and, as usual, I would have to do it all myself.

Well Diophantusy that! Those fractions look just like the one written on the board by the smart guy in the clip. You're up now, so I'm leaving. Will I see you tomorrow??

I hope not!

But remember, a mathematician is never finished with a problem. They are only finished for now. What about that other thing Marty said. He had it on the slide. *Think about it in terms of painting fifteen houses.*

I remember that, but I didn't understand what on earth he was on about. There was obviously enough trouble trying to get one house painted.

Maybe not, but now you have worked it out for yourself, you might feel less threatened by the idea of looking at someone else's way of thinking. It's like that video Max Stephens showed in the opening address of the conference.

The Abbott and Costello one.

No, the one from the classroom where the kids were telling the teacher more than one way to work out $12 - 7\,{}^{4}/_{11}$.

Who cares! It's another stupid...

The kids cared. They cared about the answer, but, more importantly, that teacher had somehow empowered them to care about working it out for themselves and wanting to explain it to the teacher. There's a role reversal!!

Okay, okay. I get it. Now go away! ... And as for you Burkard and Marty. I'm at the shop door, I haven't had a sleep in and ... I hope you two never find a film with a bath that has two plug holes and three taps!!

Keep smiling,

Doug. Williams