



"Same or Different" and other VCE appropriate tasks.

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Number Charts Multiplication in a table format Fraction estimation

Multo Walk the plank Licorice factory Factorgrams

Biggest Volume Angle estimation This goes with this

Area of a triangle Newspaper pathways

Newspaper shapes Country maps Building views Garden beds

First principles percent Four cube houses Backtracking

Rectangle fractions Chocolate cake Fraction magic squares

Protons and anti-protons Knight's tour

Circumference of a circle Pentagon triangles

Area of a circle Algebra walk

Birth month paradox

Planets Duelling dice

Pythagoras and other polygons

Baby in the car Mushroom hunt

Cylinder volumes and the Pacific Ocean

Algebra charts

Jumping kangaroos

Task	#
Addition Totals	8
Find My Pattern**	10
Lining Up	11
Heads and Legs	14
Eric the Sheep	45
Algebra Through Geometry*	71
Snail Trail	82
Square Numbers	111
Icecream Flavours*	118

Task	#
Red to Blue*	132
Painted Rods	152
4 Arm Shapes	154
Mirror Patterns 2*	159
Crossing the River 1	173
Unseen Triangles	179
Making Monuments	180
Pointy Fences*	181
Smooth Edge Tiles*	220

Year 7/8 Algebra and Pattern

16. Octaflex
31. Cube Nets
36. Making Triangles
44. Latin Squares
66. Pentominoes
67. Making Solids
68. Six Square Puzzle
70. Symmetric Shapes
77. Tricube Constructions A

90. Tricube Constructions B
95. Reflections
150. Chess Queens
153. Knight Protectors
155. $64 = 65$
157. Paving Views
166. Sphinx
237. Trisquares
238. Growing Trisquares

Year 8 Space & Geometry

Task	#
Dice Differences	34
Cat And Mouse	223
Final Eight*	1
First Down The Mountain	87
Have A Hexagon	53
Same Or Different	18
What's In The Bag?	198

Task	#
Choosing Beads	191
Counter Escape	6
Highest Number 2**	128
Greedy Pig	200
Duelling Dice	46
Dice Footy*	232

Year 9 Chance

Task	#
Can Stack **	27
Cube Numbers**	208
Difference Between Two Squares **	64
Double Staircase	61
Fold Up Houses **	55
How Many Triangles? **	48
Intersections **	206
Money, Money, Money **	233
Painted Cubes	160
Pyramid Puzzle	101

Task	#
Time for Tiling	140
Squares Around Squares **	24
Tetrahedron Triangles **	186
Tower of Hanoi	142
Triangles and Colours *	221
Triangles around Triangles **	42
Two Squares **	57
Star Numbers **	236
Smooth Edge Tiles **	220

Year 10 Algebra

Today we will touch on:

Gradient Functions

Making Monuments

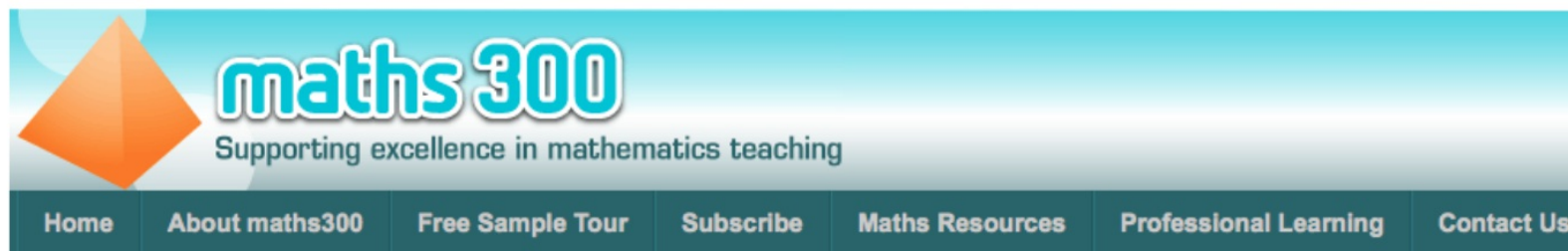
Biggest Volume

Farmyard Friends

Trigonometry Walk

Same or Different

Gradient Functions



Lesson 136

Gradient Functions

[Overview](#) | [Lesson Plan](#) | [Classroom Contributions](#)

Lesson Plan

Years: 10 - 12

Time: 2 - 4 lessons

Strands: Measurement, Number, Space

Related worksheets

- [Investigation Sheet](#)
- [Recording Sheet](#)

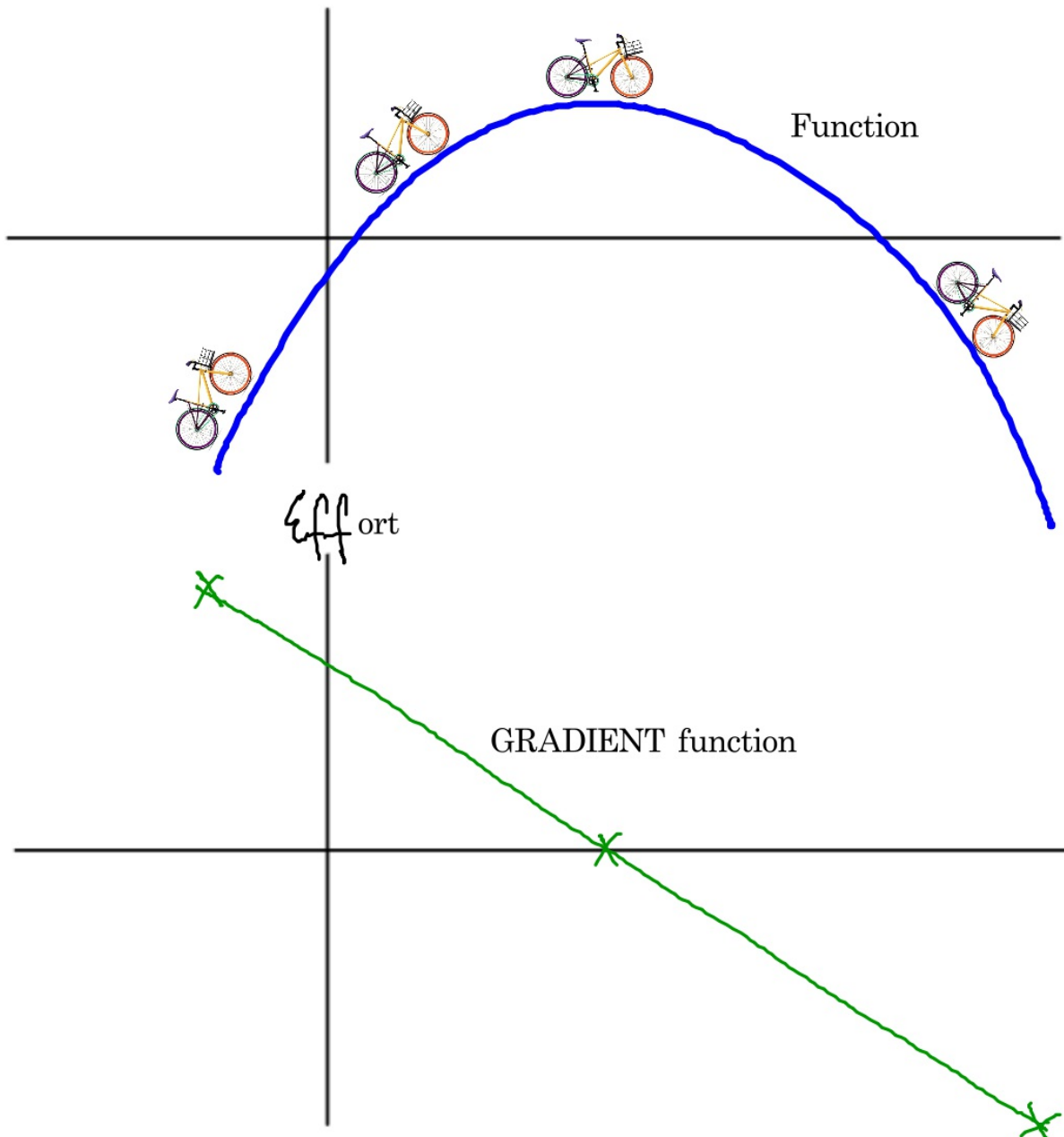
Summary

This lesson explores the concept of the gradient function, that is, the function that describes the way the gradient changes.

Having focused on the idea of the gradient of a function through their own experiences with riding a bicycle up and down hills, students are invited to explore the relationship between the x -value of the function and the gradient at the point (x, y) .

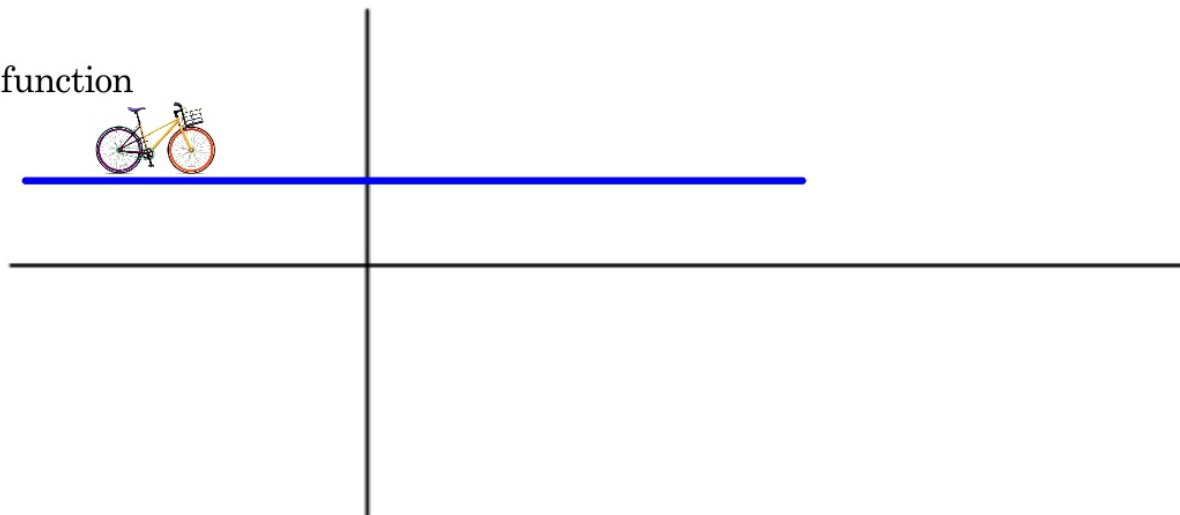
Secondary goals include:

- a growing ability to visualise the shape of a gradient function just by looking at a graph
- recognition of the types of functions that gradient functions can be
- how this is related to the type of initial function
- awareness that continuity is an important feature of a graph for a gradient function to exist.

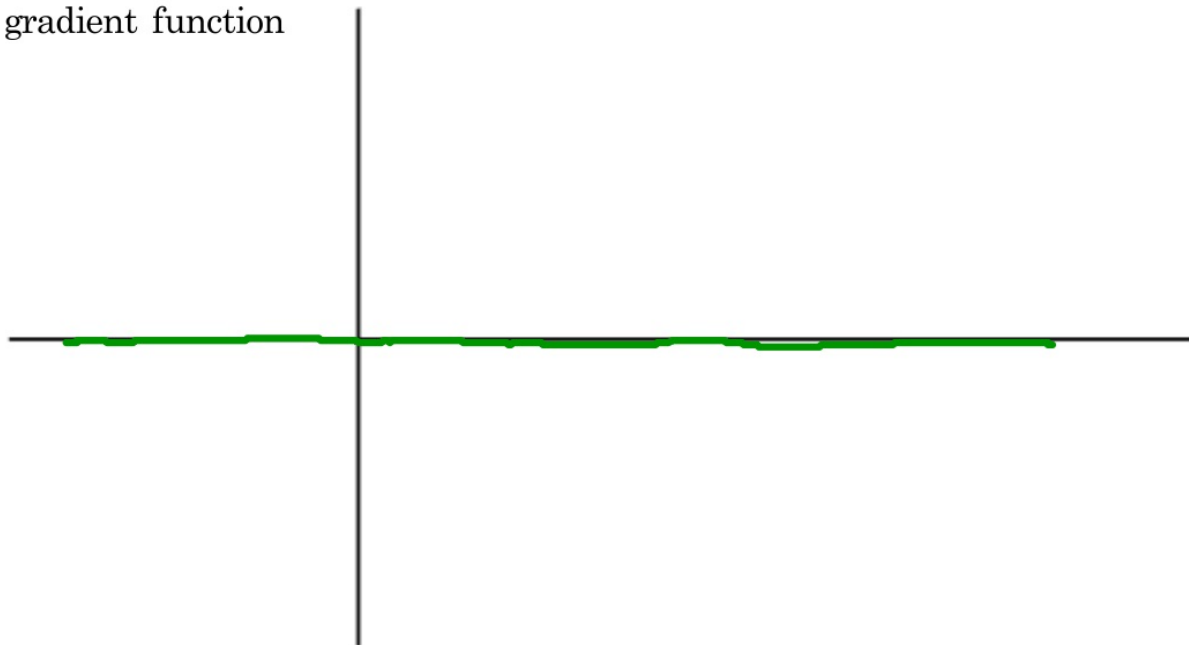


Relating the gradient of a function to the effort required to ride a bicycle over the function.

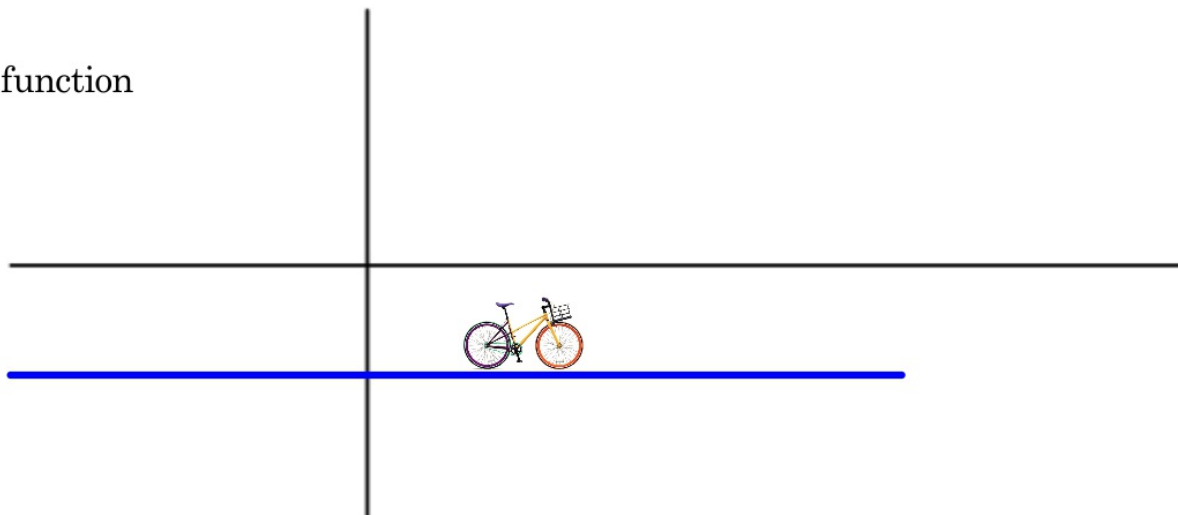
function



gradient function



function



gradient function



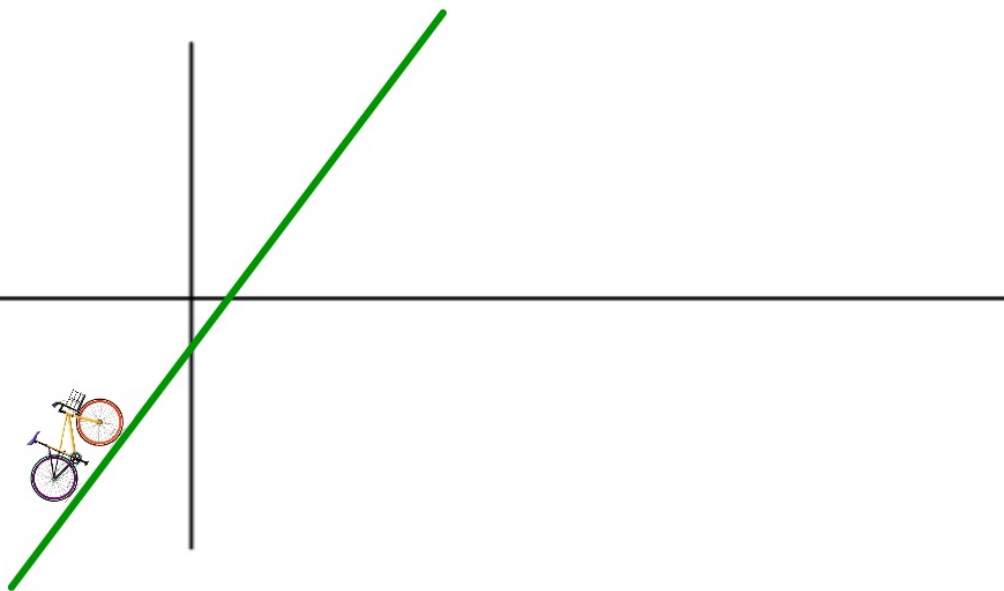
function



gradient function



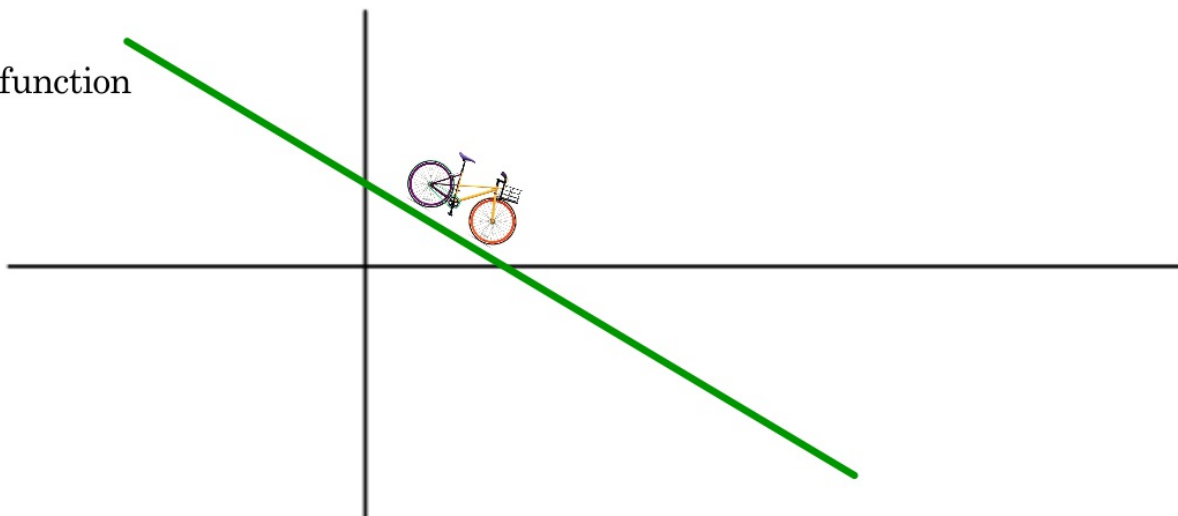
function



gradient function

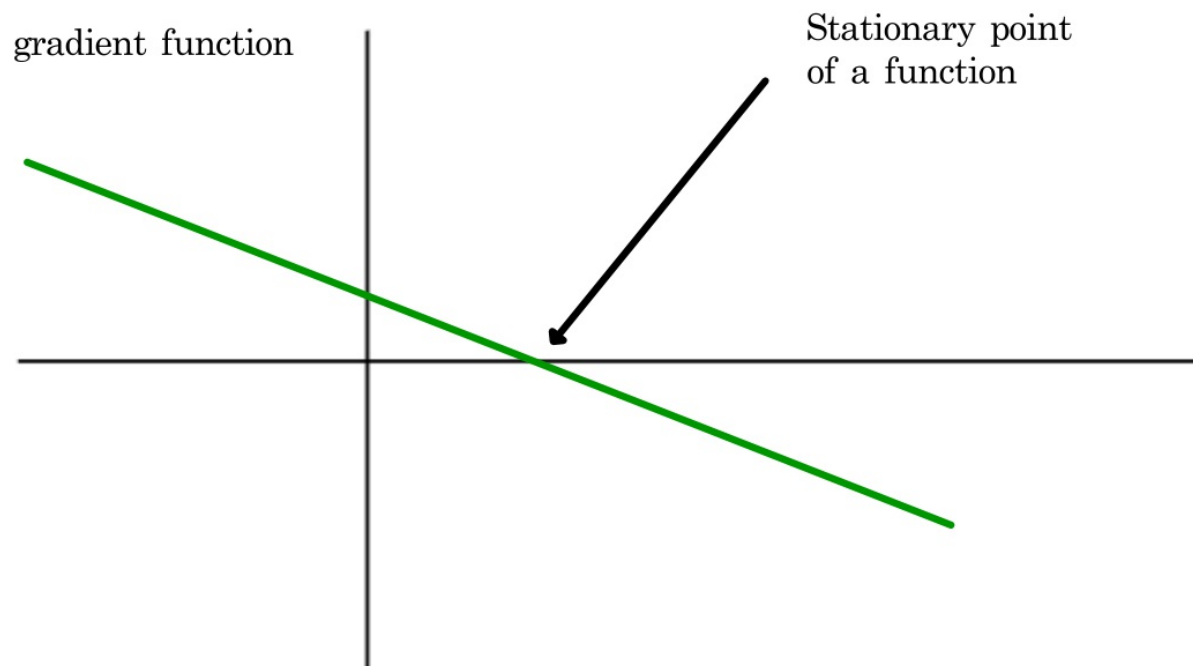
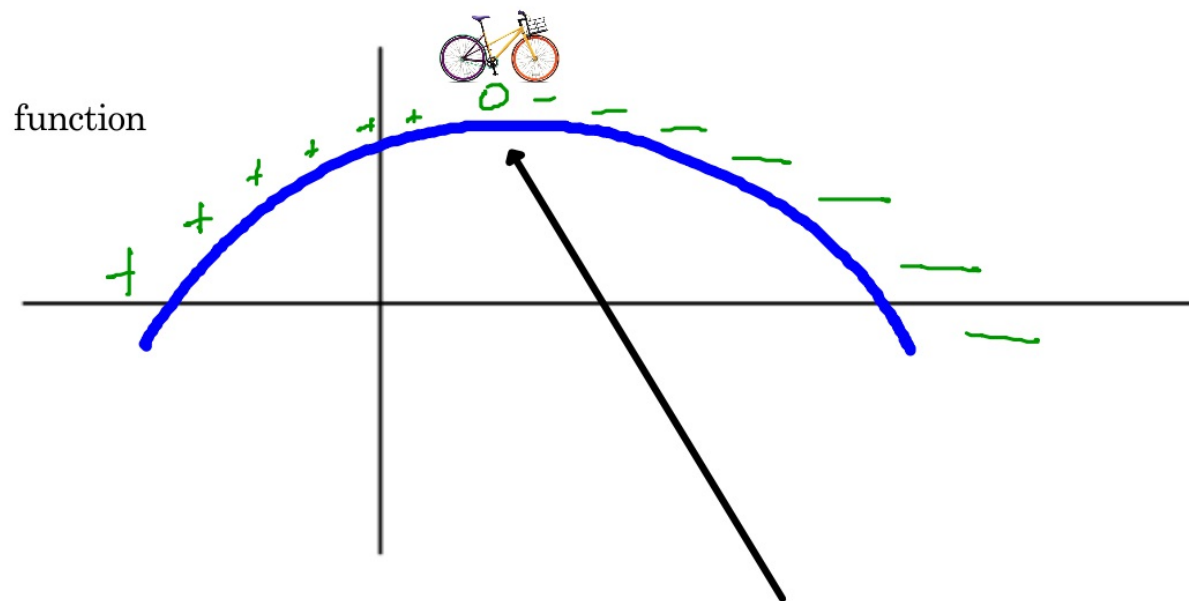


function



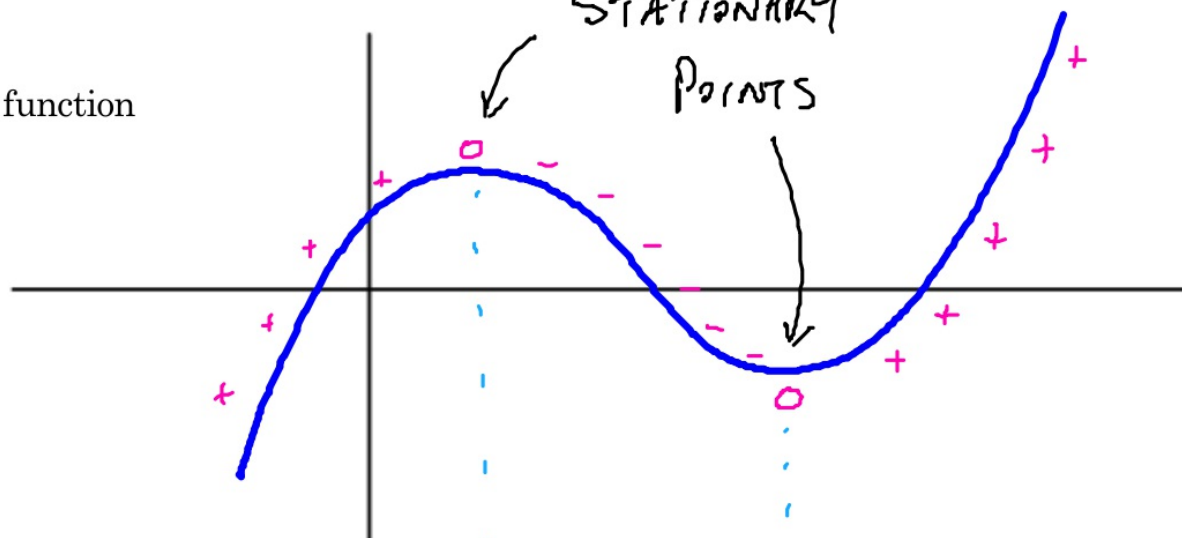
gradient function



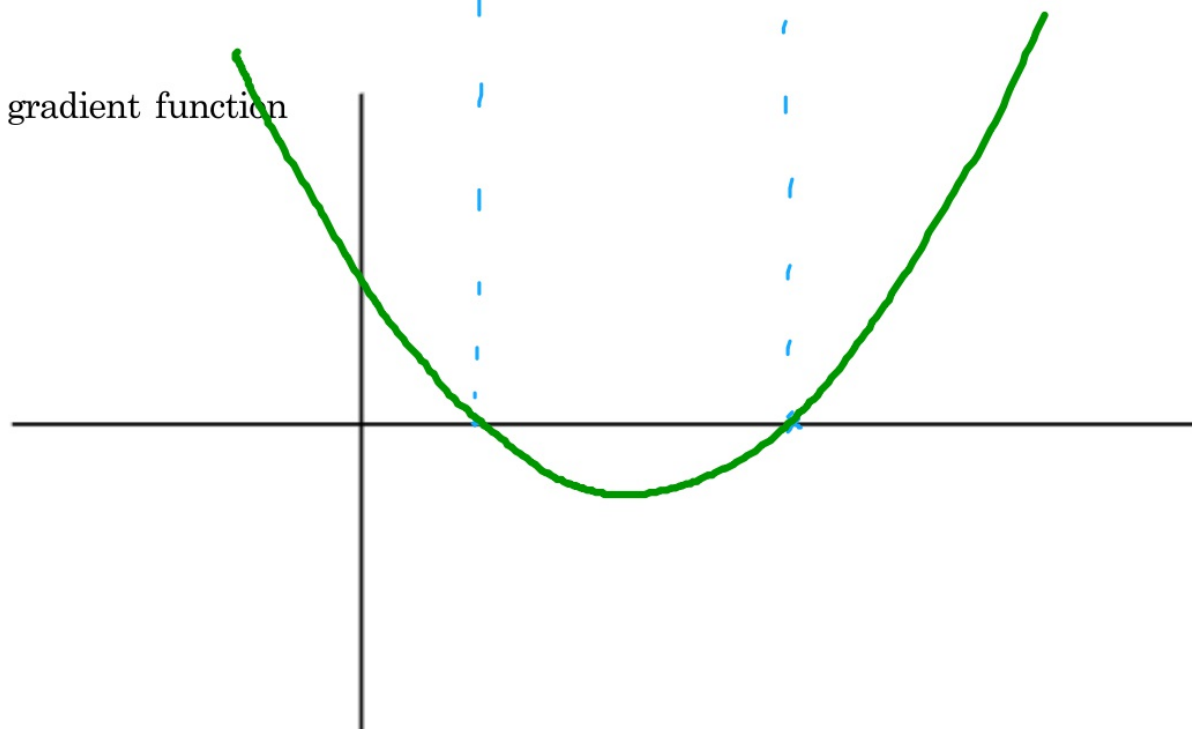


function

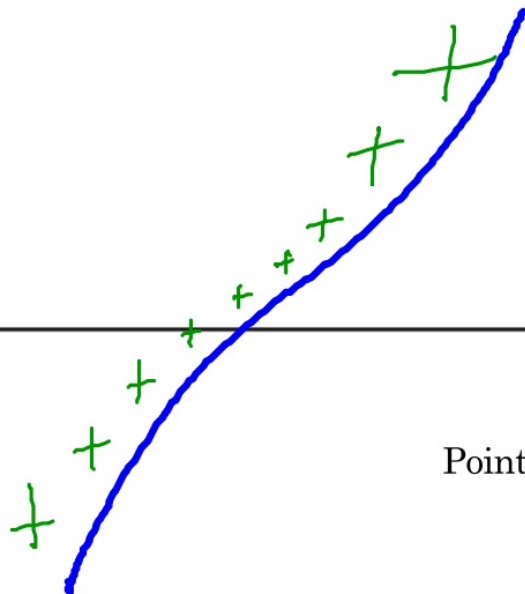
STATIONARY
POINTS



gradient function



function

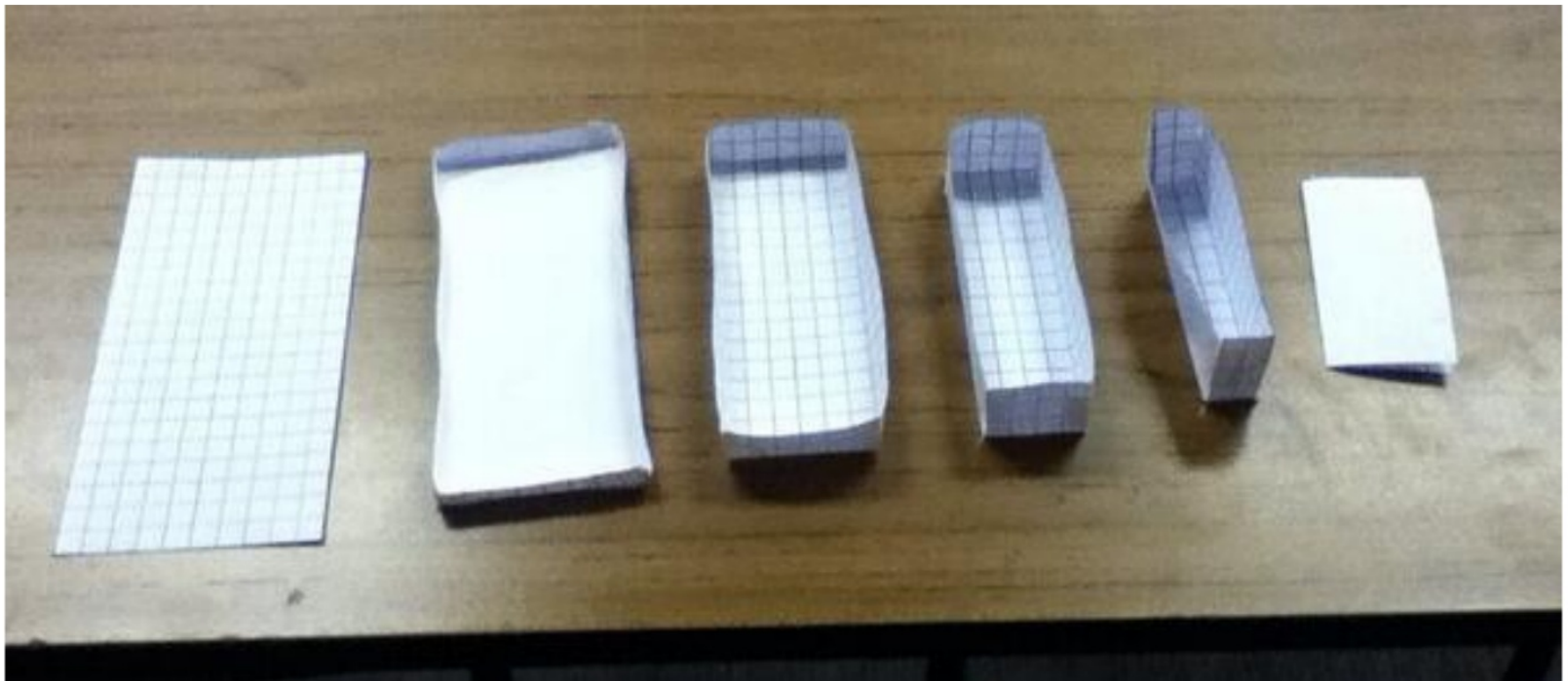


Point of inflection (but not stationary)

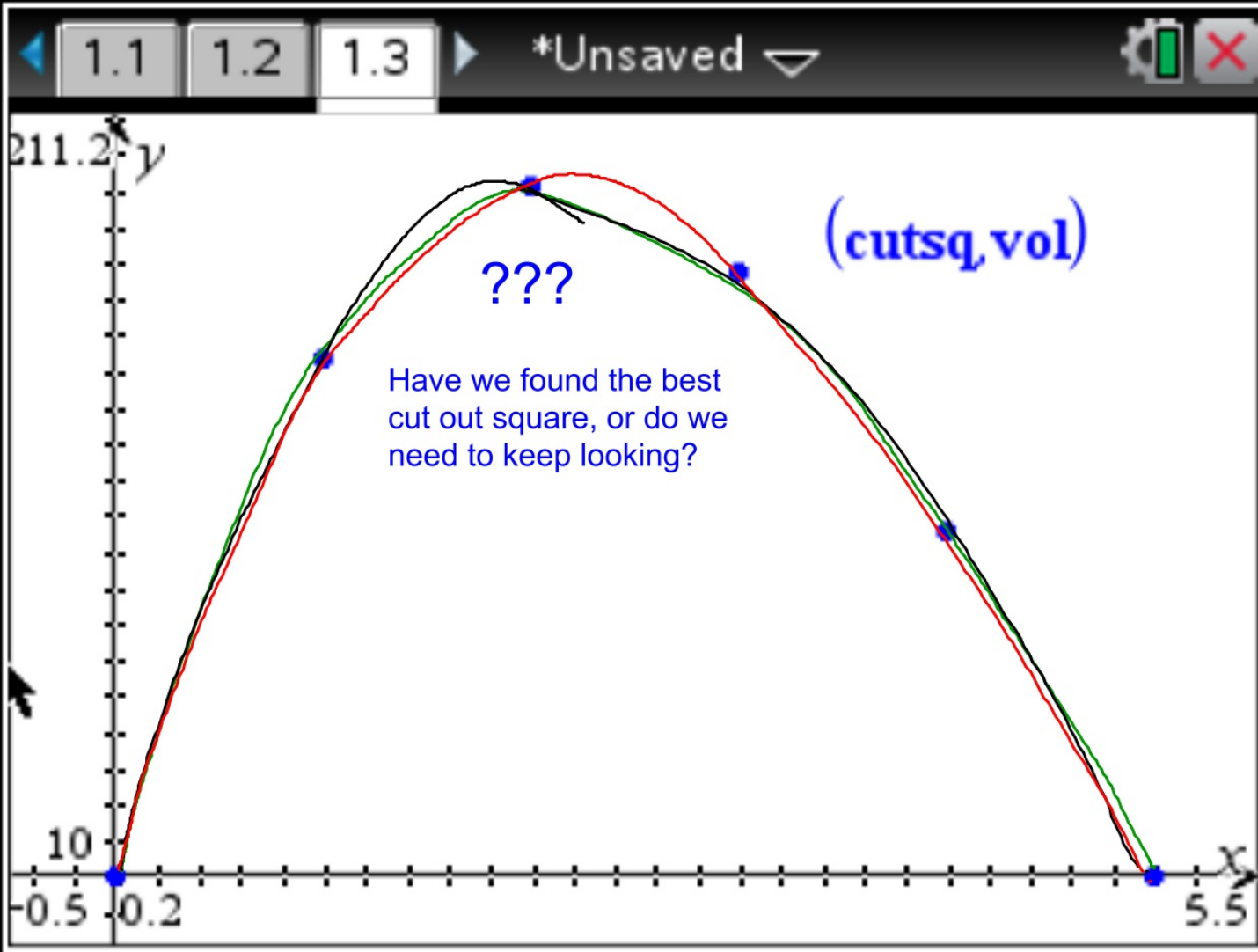
gradient function



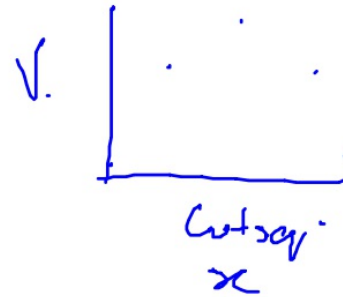
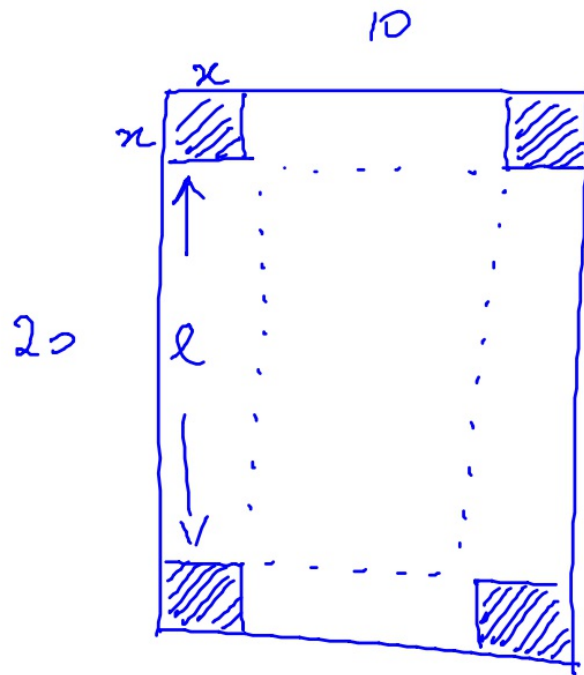
Which has the biggest volume?



Cut out Sq	L	W	H	V
0	20	10	0	0
1	18	8	1	144
2	16	6	2	192
3	14	4	3	168
4	12	2	4	96.
5	10	0	5	0



Setting the problem up to use CALCULUS



$$V = l \times w \times h$$

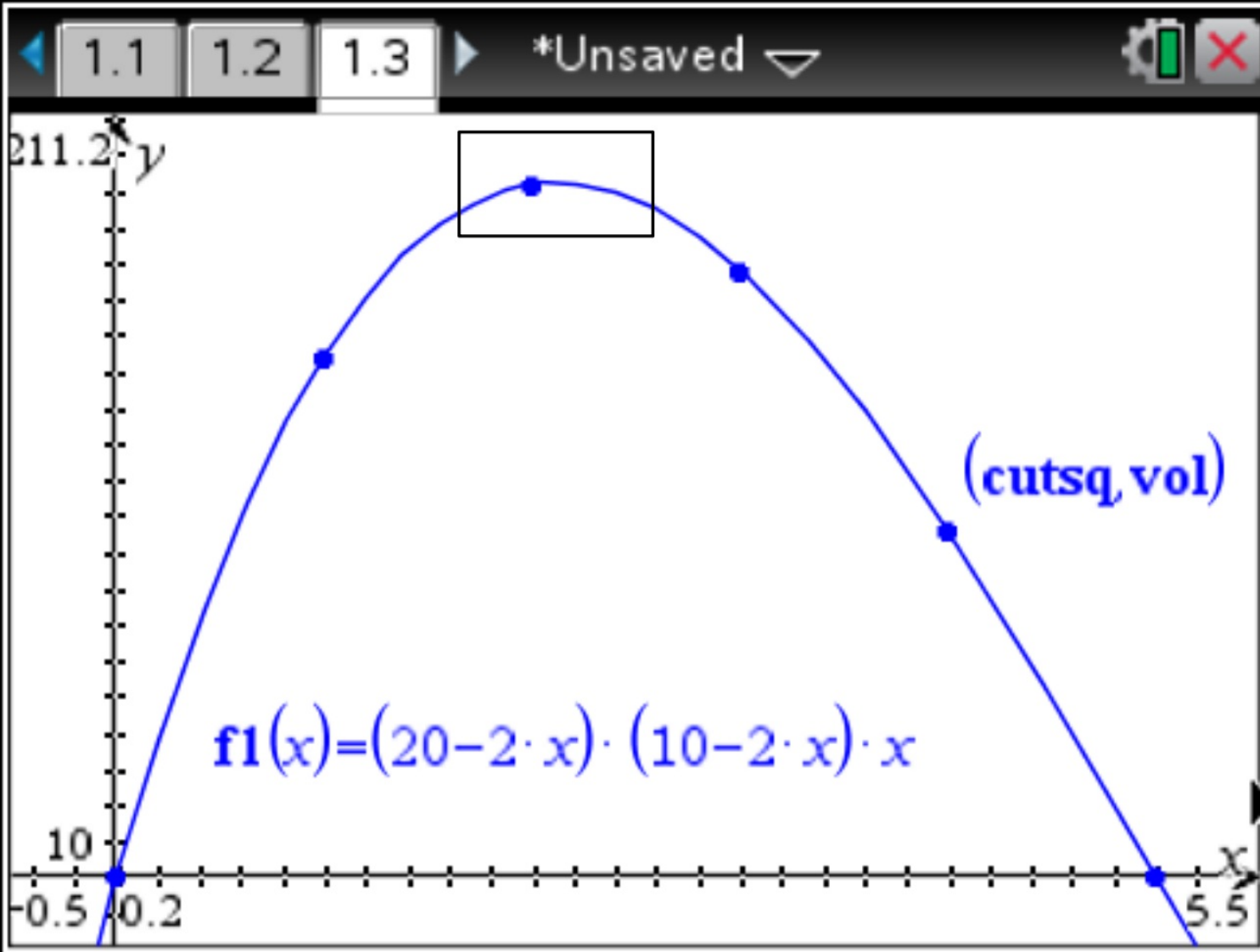
$$l = 20 - 2x$$

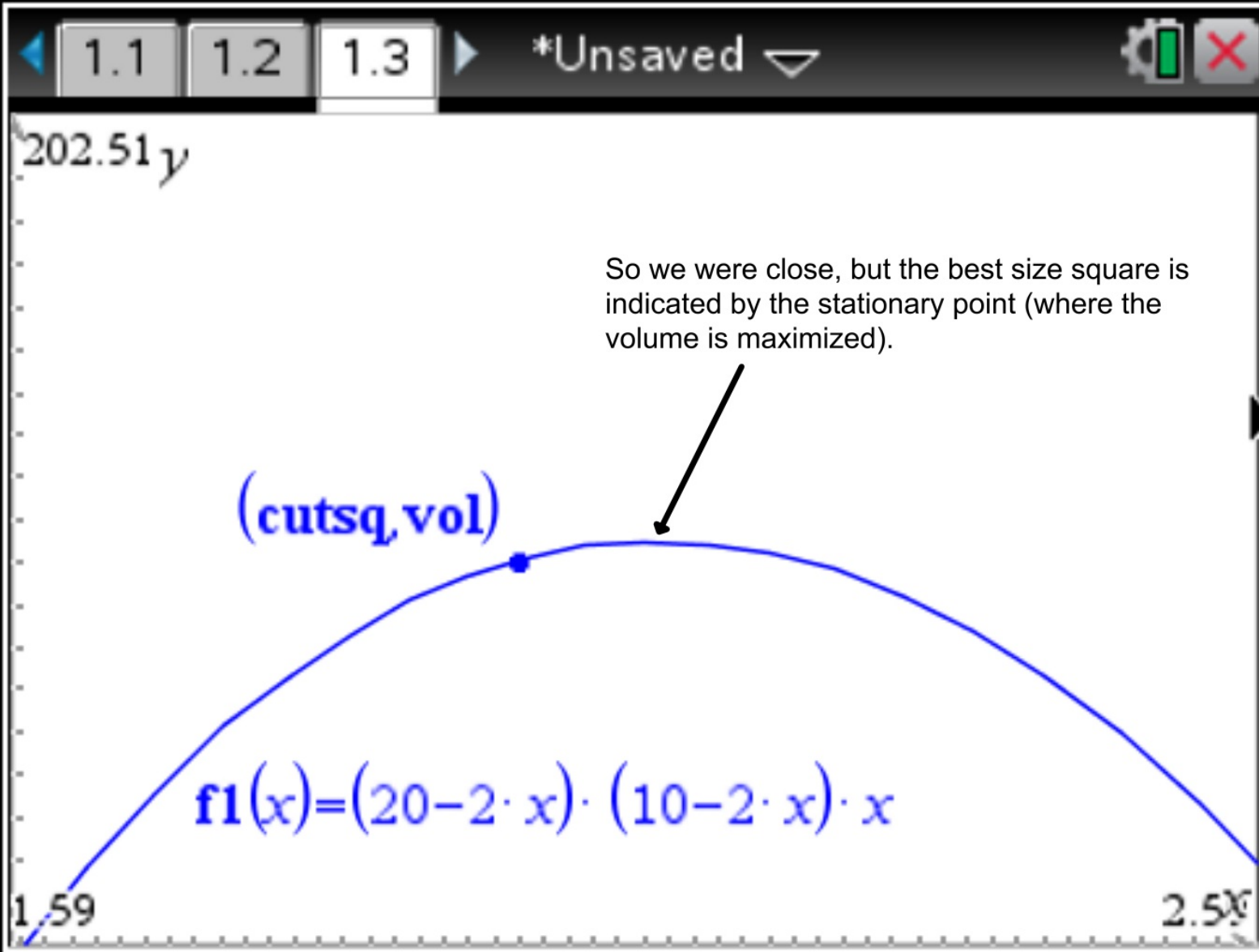
$$w = 10 - 2x$$

$$h = x$$

$$V(x) = (20 - 2x)(10 - 2x)x$$

A function for the volume where x is size of the cut-out square





Calculus will let us find the exact values of the stationary point

Solve $V'(x) = 0$ for x (ie find the stationary point).

$$\begin{aligned} V(x) &= (20 - 2x)(10 - 2x)x \\ &= (200 - 60x + 4x^2)(x) \\ &= 200x - 60x^2 + 4x^3 \end{aligned}$$

$$V'(x) = 200 - 120x + 12x^2$$

$$\begin{aligned} \text{Let } 12x^2 - 120x + 200 &= 0 \\ 3x^2 - 30x + 50 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{30 \pm \sqrt{900 - 4(3)(50)}}{2(3)} \\ &= \frac{30 \pm \sqrt{300}}{6} \end{aligned}$$

$$x = \frac{30 \pm 10\sqrt{3}}{6}$$

$$= \frac{15 \pm 5\sqrt{3}}{3}$$

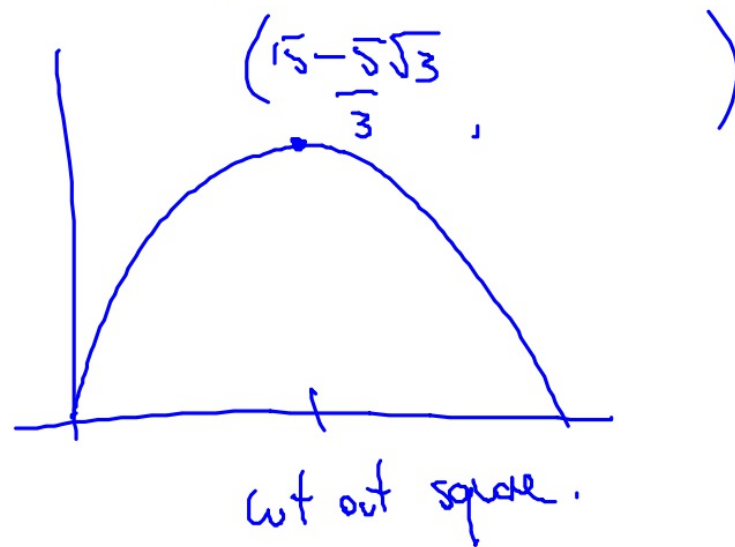
$$= \frac{15 - 5\sqrt{3}}{3}$$

the optimal value of x

(since $\frac{15 + 5\sqrt{3}}{3}$ is too big i.e. > 5)

The solution for the optimum
cut out square is $\frac{15-5\sqrt{3}}{3}$

This approximately 2.11325

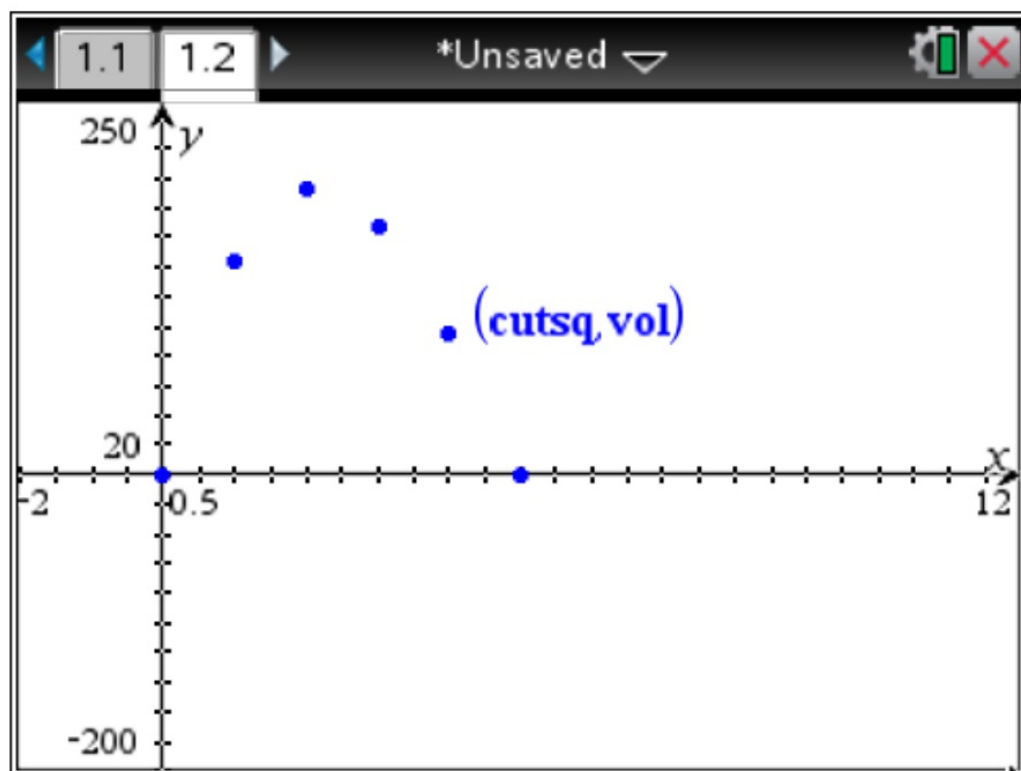


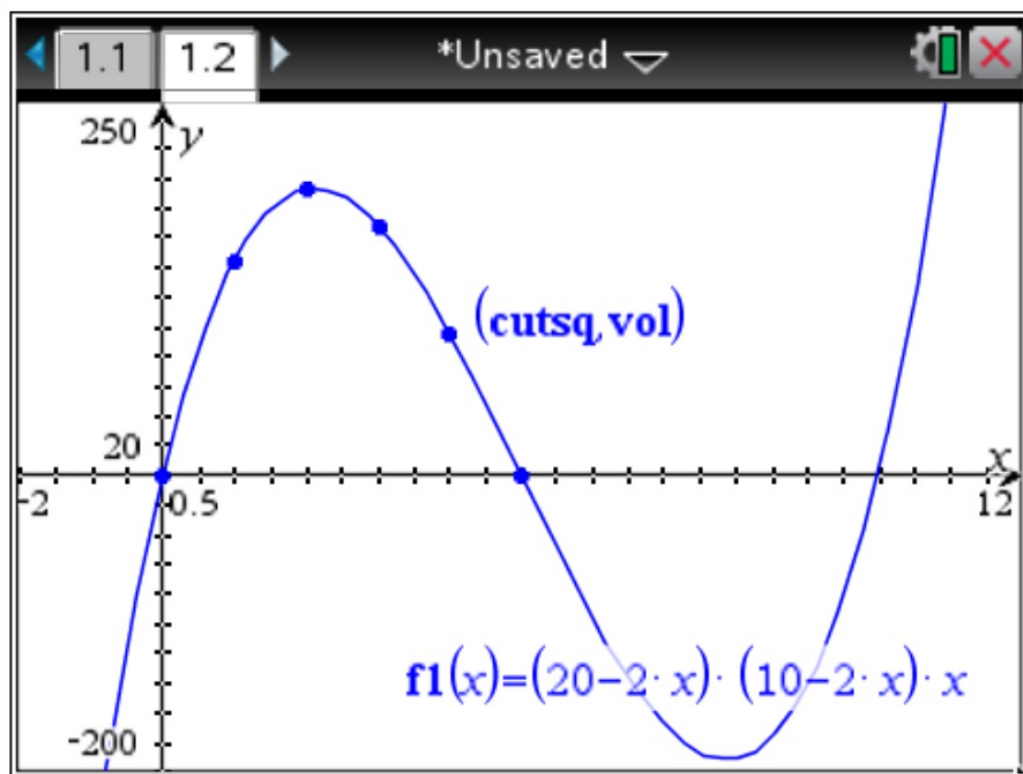
Solving the problem using CAS calculator

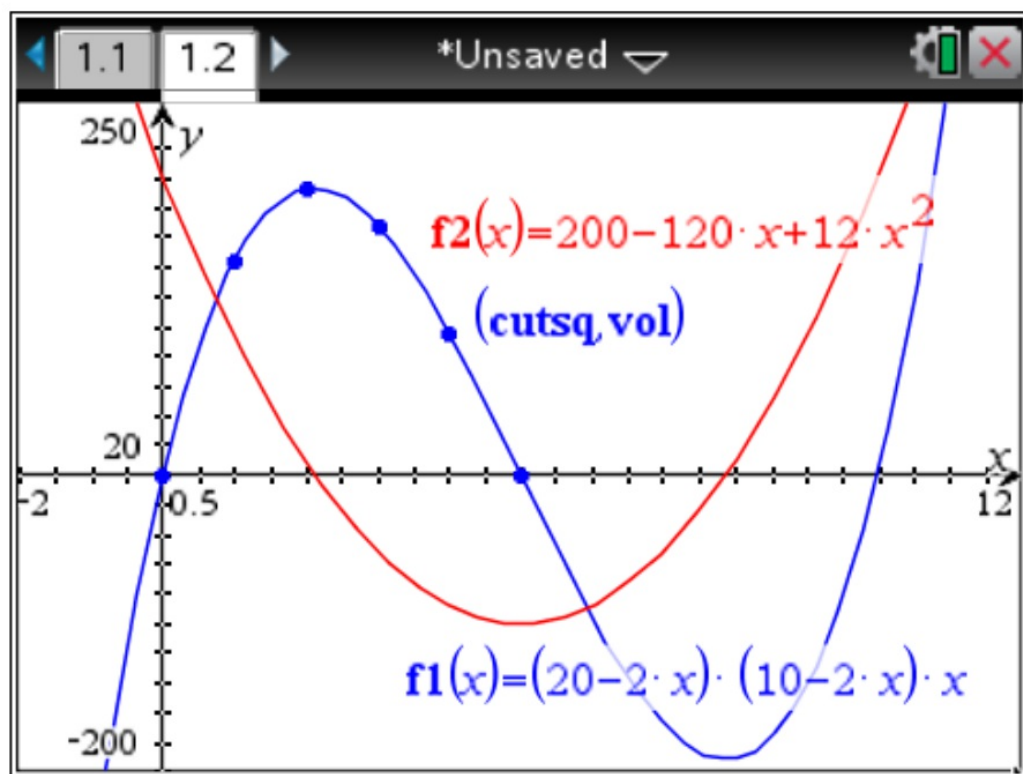
The screenshot shows a CAS calculator interface with the following content:

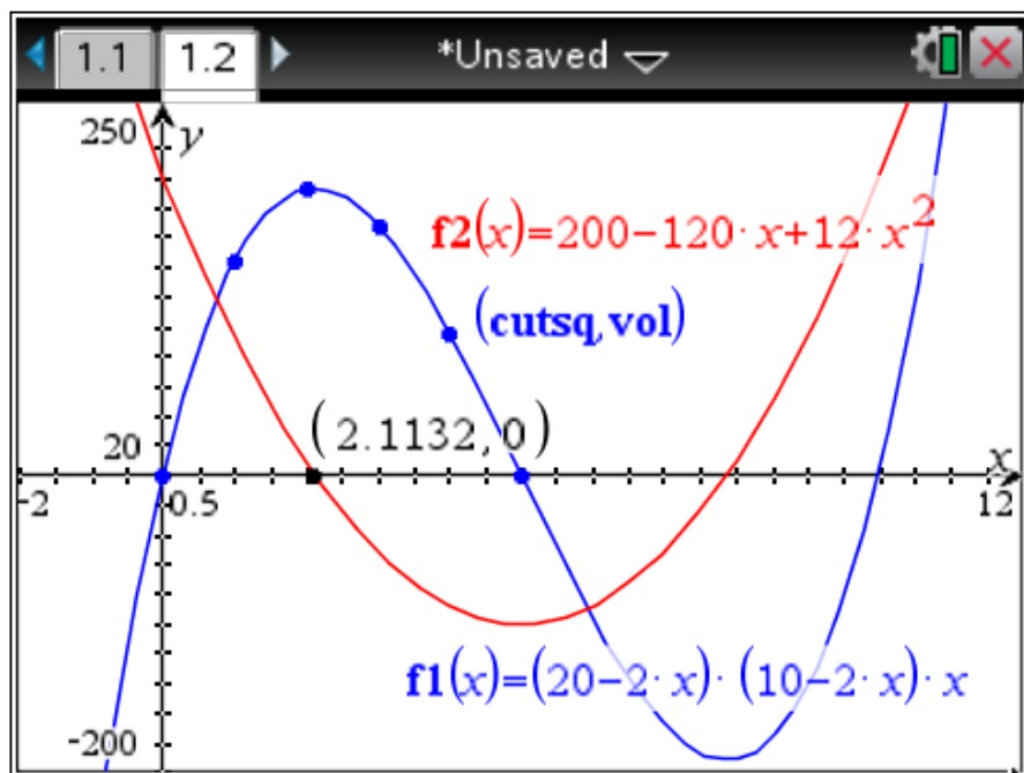
- Top bar: Navigation buttons for 1.2, 1.3, and 1.4, a status indicator for '*Unsaved', and a close button.
- Input area: The expression $\text{Define } v(x) = (20 - 2 \cdot x) \cdot (10 - 2 \cdot x) \cdot x$ followed by the word 'Done'.
- Derivative: The derivative $\frac{d}{dx}(v(x))$ is calculated as $12 \cdot x^2 - 120 \cdot x + 200$.
- Solve command: The command $\text{solve}(12 \cdot x^2 - 120 \cdot x + 200 = 0, x)$ is entered.
- Exact solution: The calculator returns the exact solutions $x = \frac{-5 \cdot (\sqrt{3} - 3)}{3}$ or $x = \frac{5 \cdot (\sqrt{3} + 3)}{3}$. A red arrow points to the first solution.
- Numerical solution: The same command is repeated, and the calculator returns the numerical solutions $x = 2.11325$ or $x = 7.88675$. A red arrow points to the first numerical solution.
- Bottom bar: A cursor is visible in the input field.

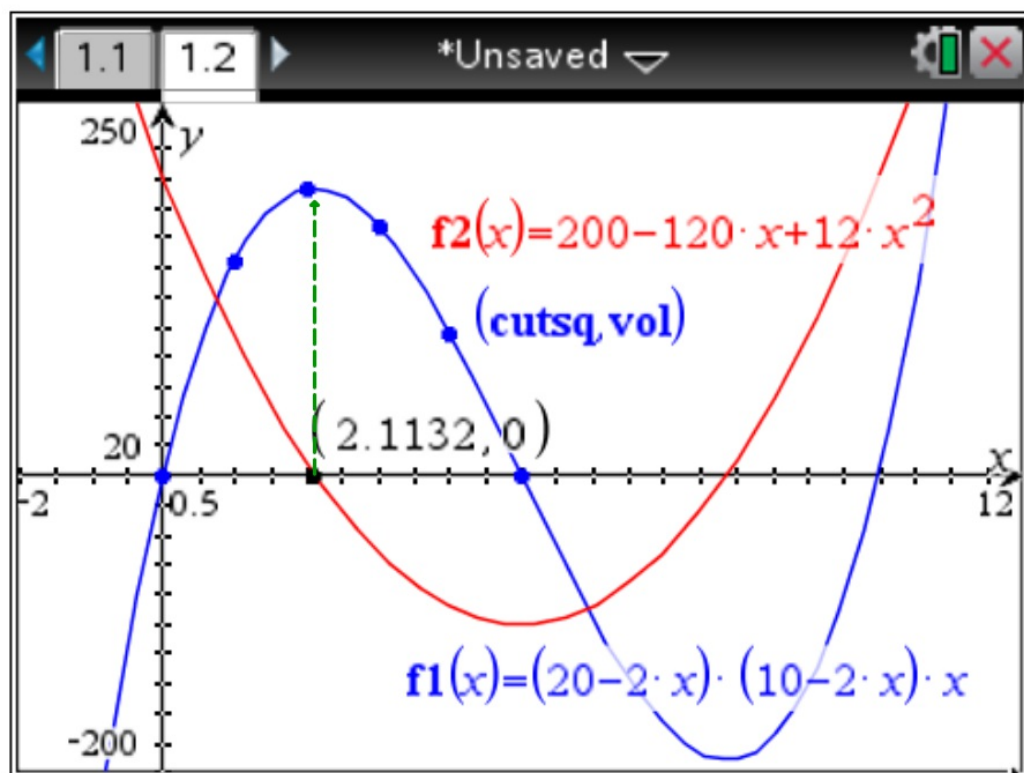
The same concept, the calculator just does all of the "number-crunching".

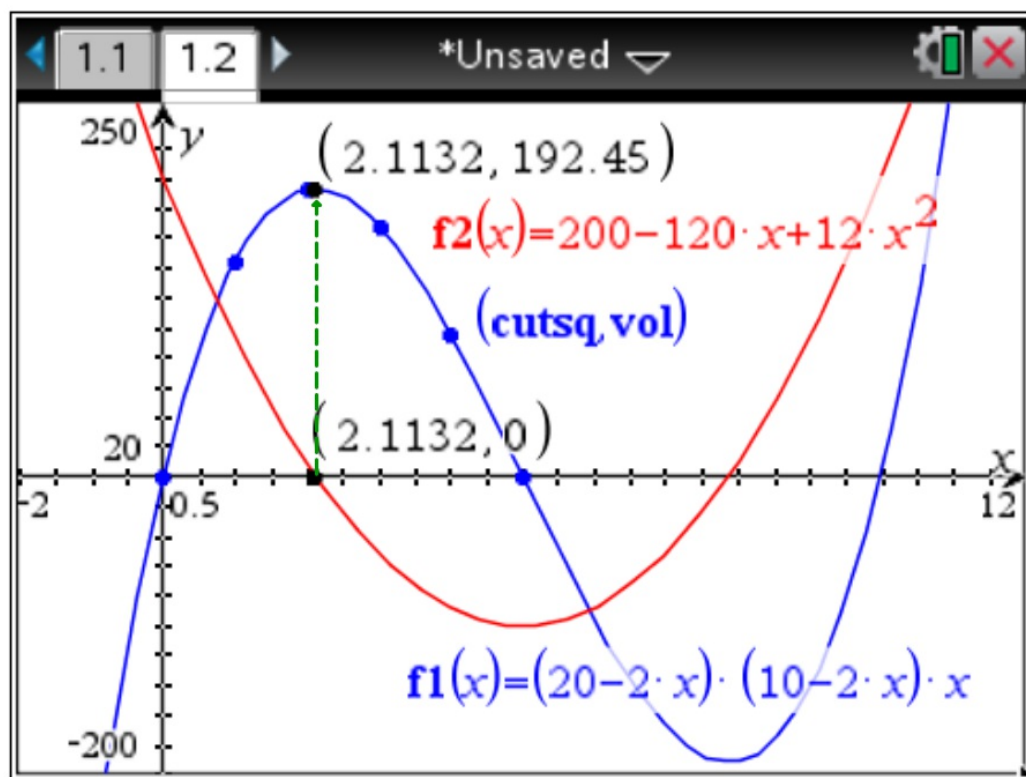






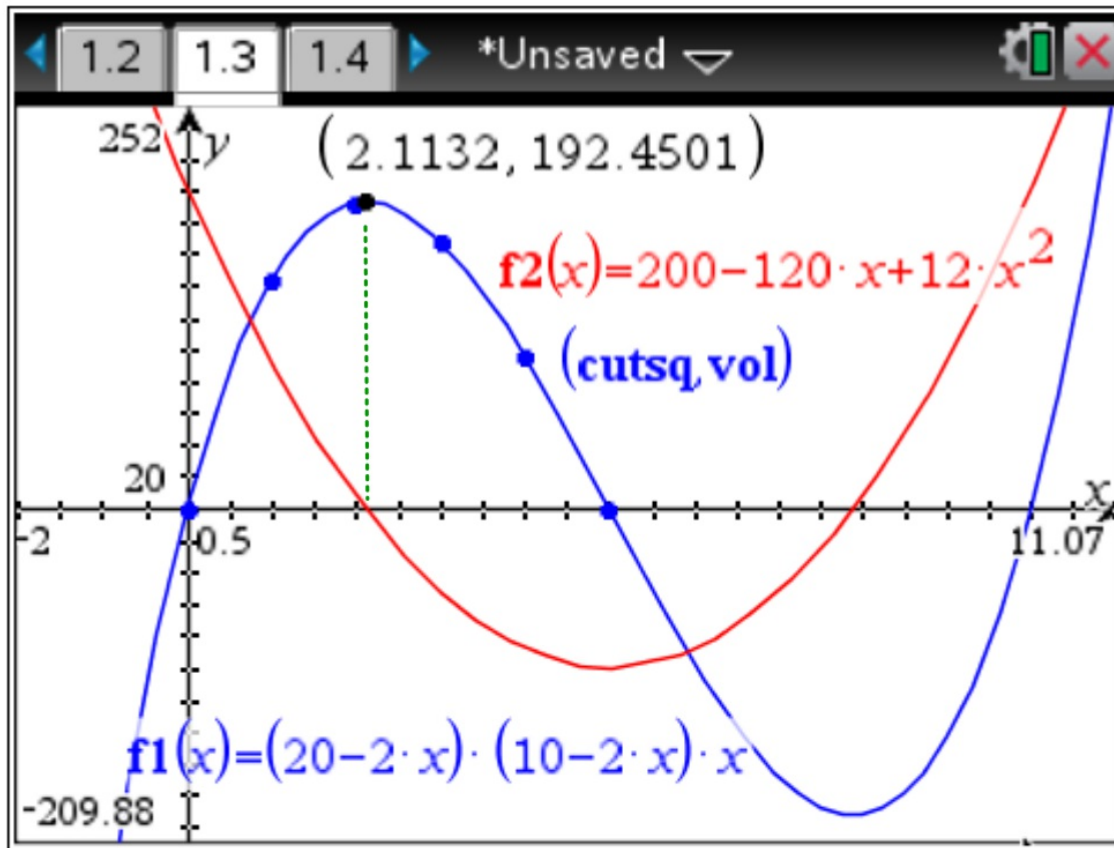






The "Big Picture"

Here we can see



- the initial volumes we found,
- the volume function we defined,
- the gradient function,
- the solution where the gradient function is zero,
- the maximum volume of the cuboid.

Trigonometry Walk



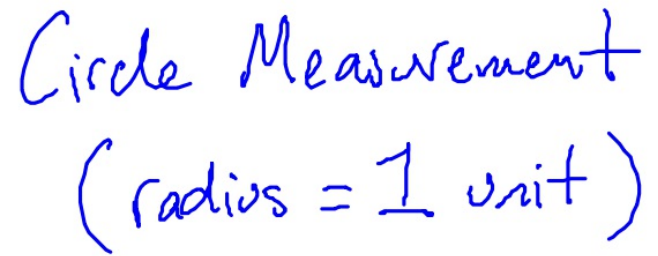
Damian Howison @DaHowi · 2h

Waggin' class? Nah, doin some trig-walkin [#math](#) [#maths300](#) [#mathmethods](#)

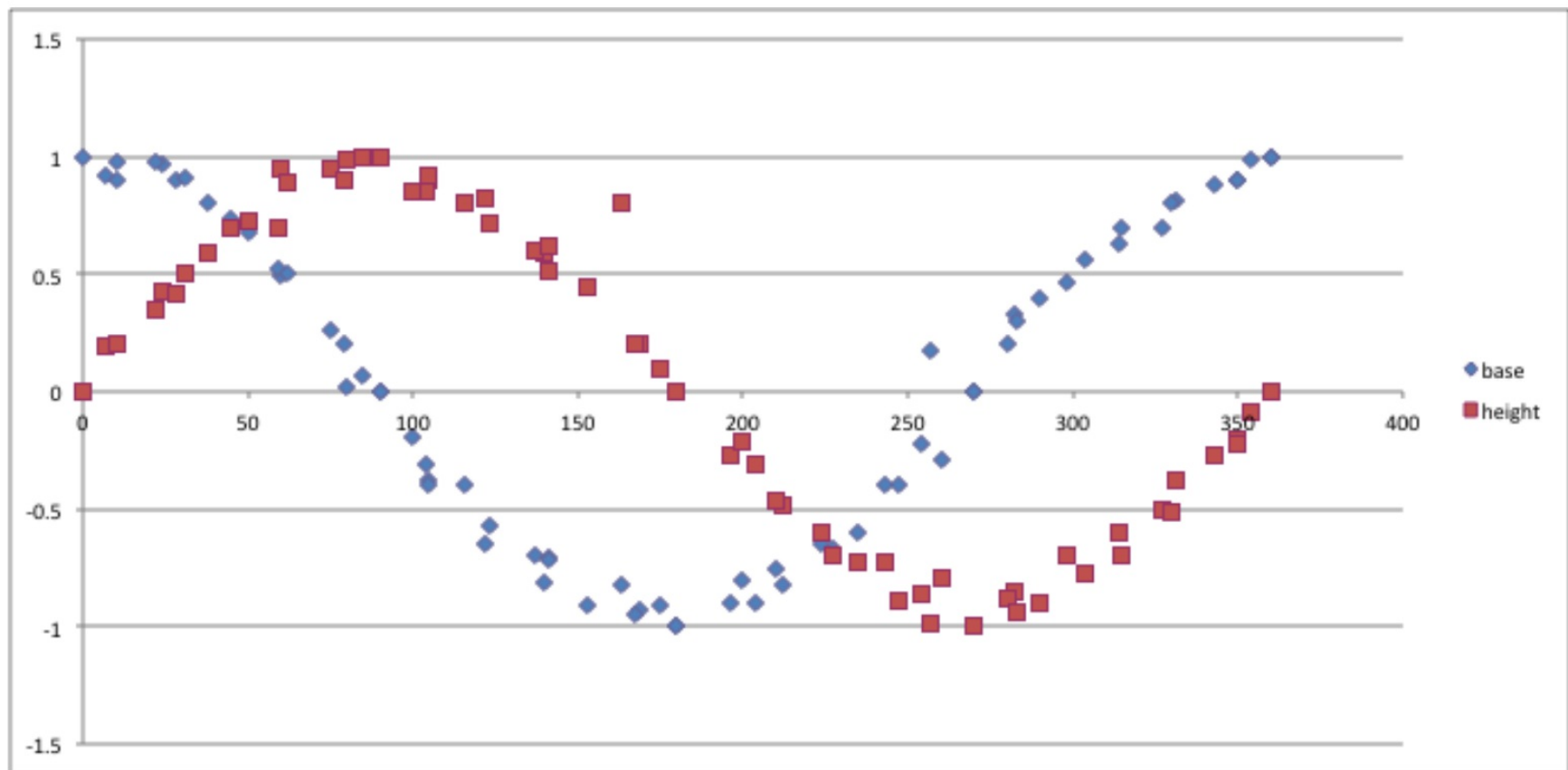


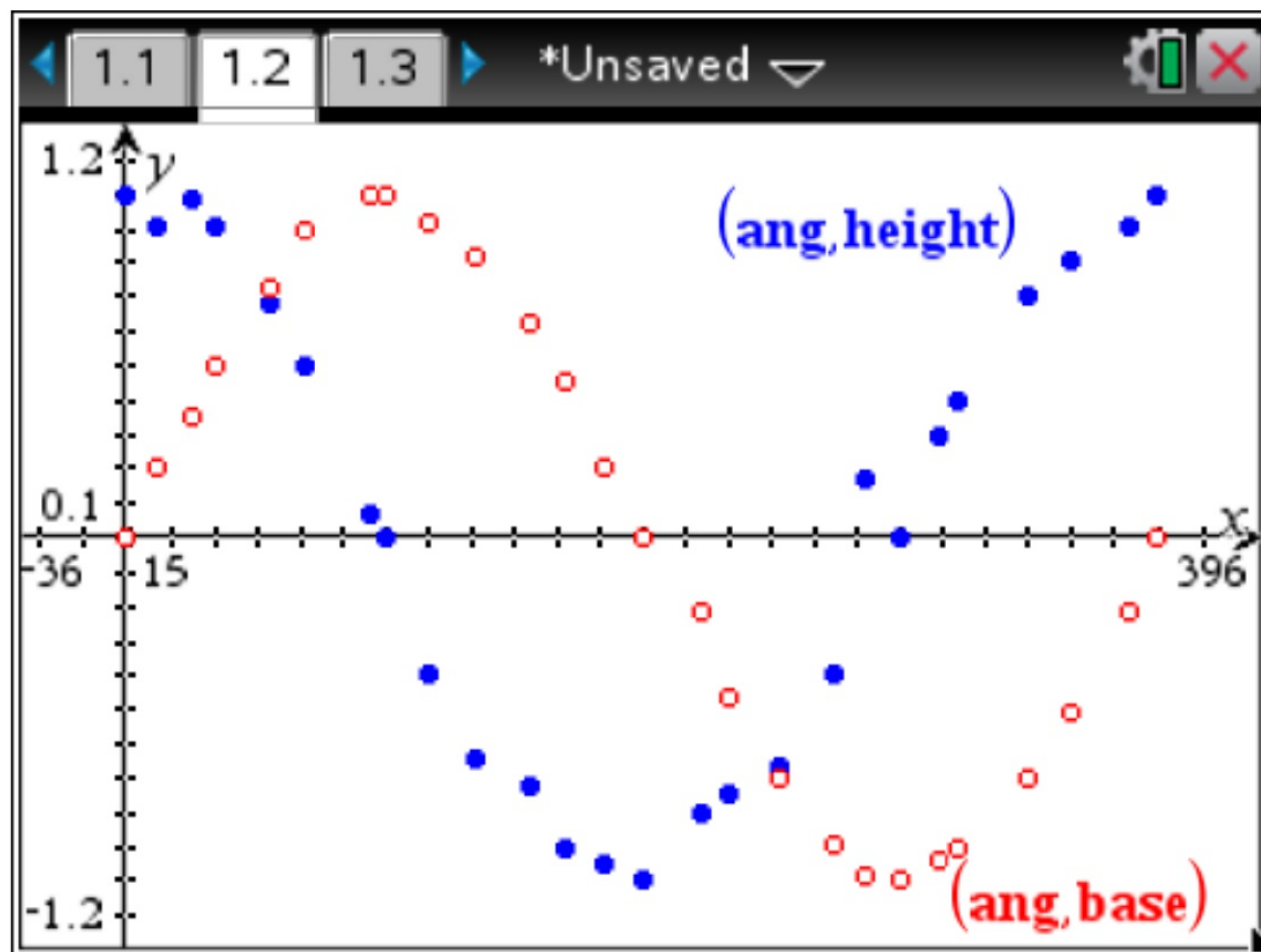
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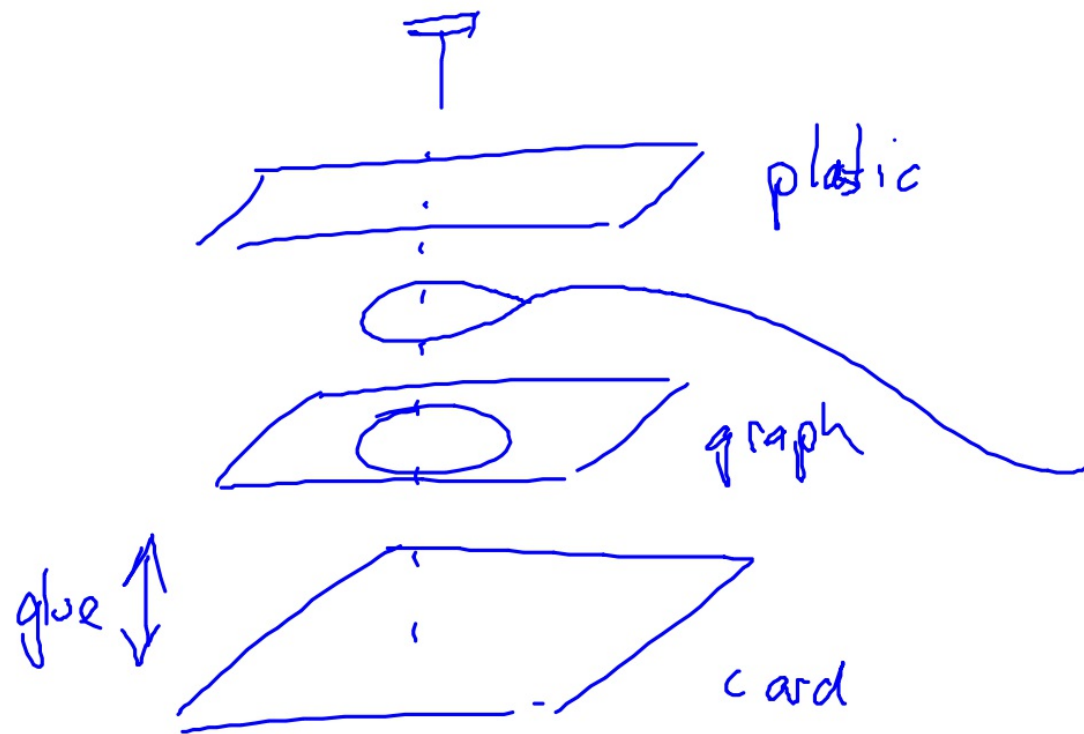
 Reply  Favorite  More

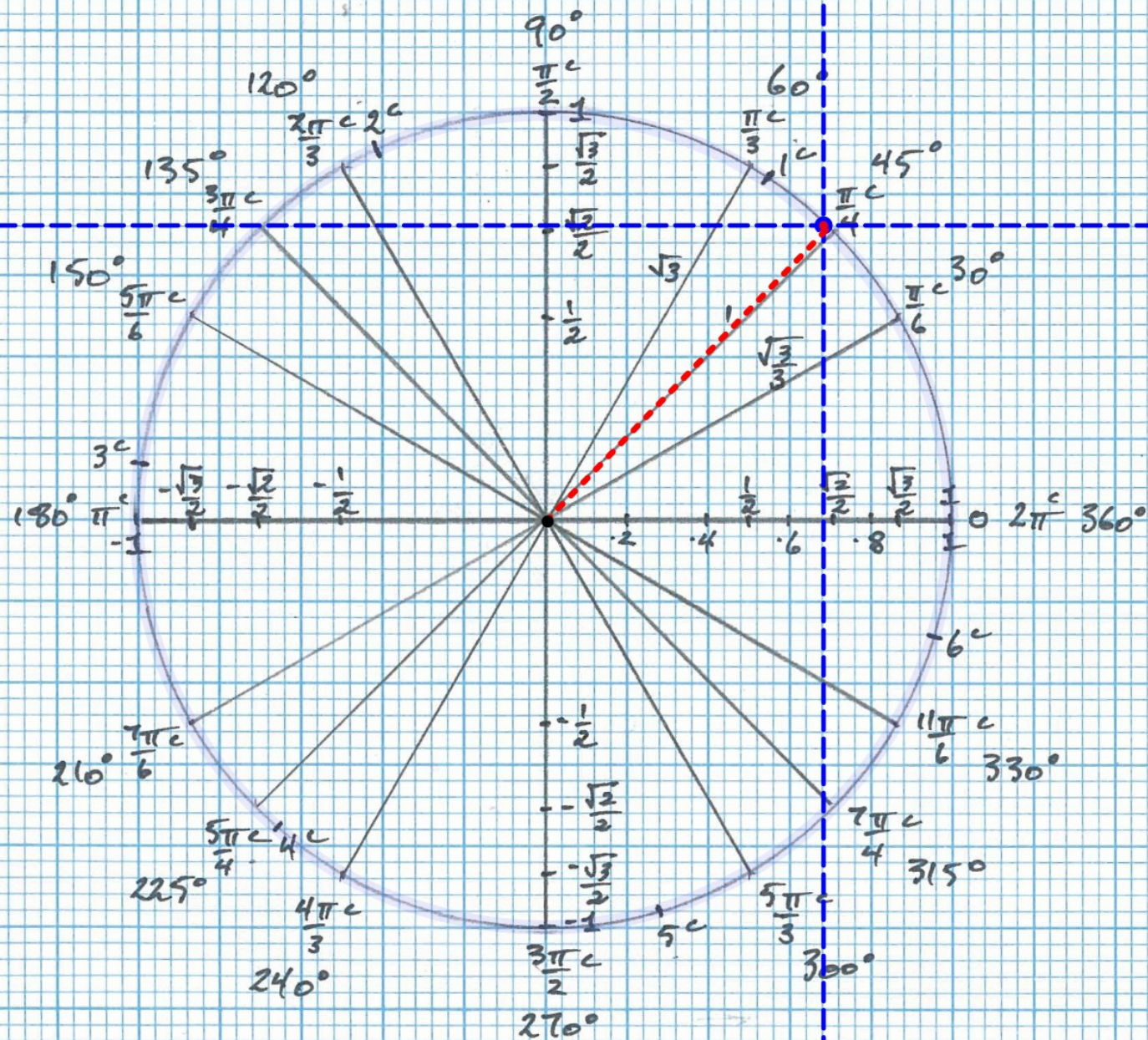


Angle	Height	Base

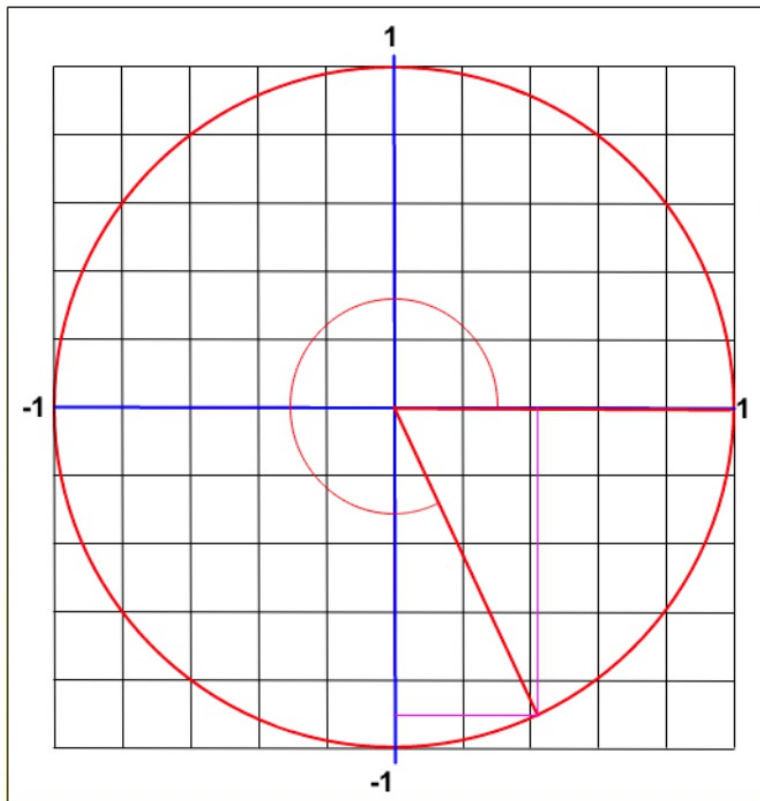








Intro to solving trig equations



A = 295 degrees

Click on the circle.

Use the unit circle to estimate both angles (within 1 degree accuracy) for the following trigonometric ratios.

Ratio **angle 1** **angle 2**

$\tan A = -1$		
$\sin A = -0.9$	245	295
$\cos A = 0.3$		
$\cos A = -0.8$		
$\sin A = 0.3$		

244 296

OK

Answers

You Need

- Five [5] 2cm cubes and twenty [20] 2cm tiles and a recording sheet

The Story

In the parks of Slovenka they build monuments from cubes. Then they make four [4] paths that are the same length as the height of the monument. This is a Size 2 monument and paths. The monument and the paths are tiled with the same size tiles as one face of the cube.



Your Task

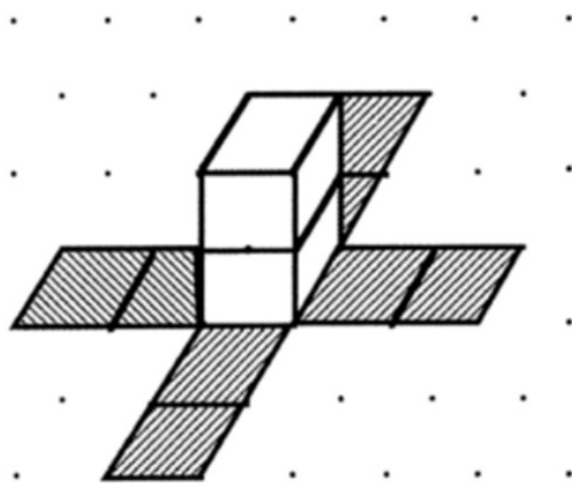
1. Make the Size 1 monument and paths. How many tiles would be needed?
2. Make other size monuments and paths up to Size 5 and work out how many tiles would be needed in each case.
3. Record the answers to Questions 1 and 2 in your journal.
(A mathematician might make a table to record this information.)
4. How many tiles are needed to make a Size 10 monument and paths?

Challenge

How many tiles are needed to make a Size 100 monument and paths?
If I tell you any size monument explain how to calculate the number of tiles.
What happens if I don't understand your explanation?
Find at least one other way to explain it to me.

Making Monuments: Investigation Sheet No.1

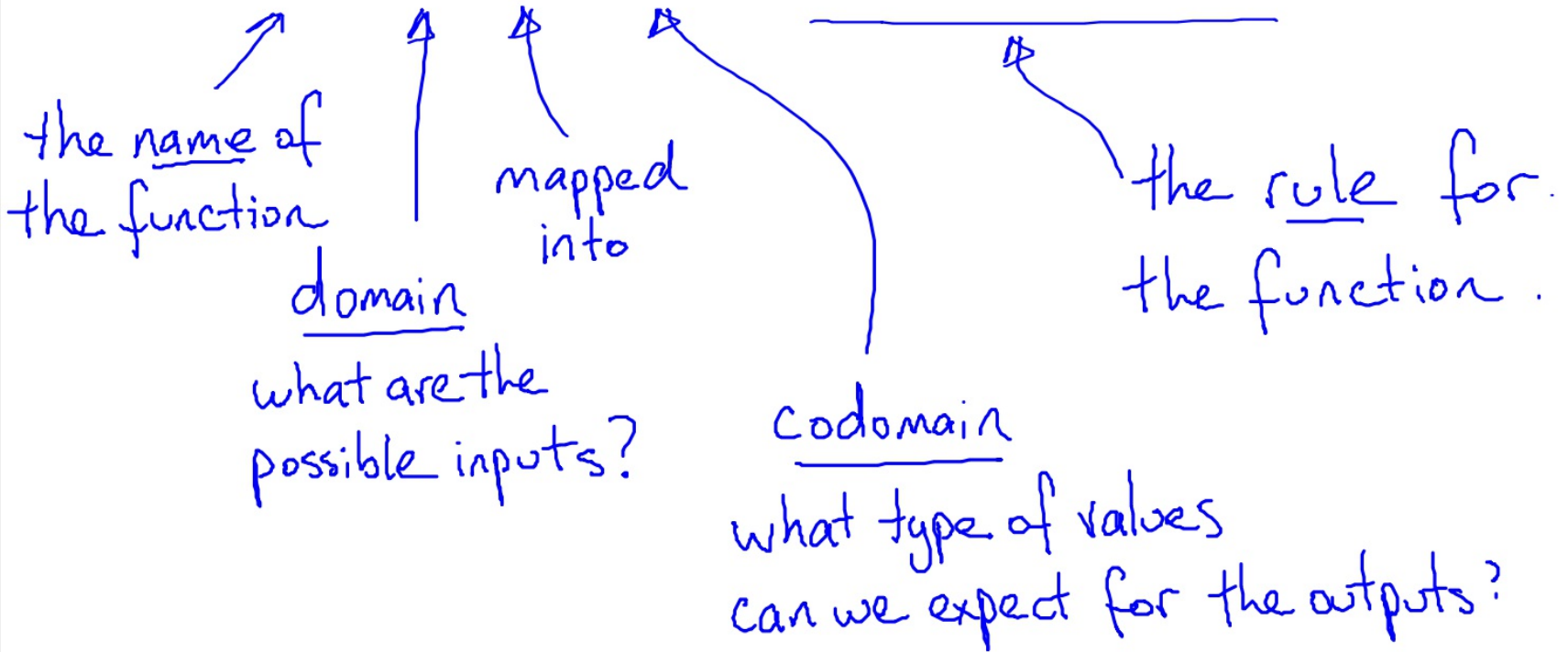
Monuments are scattered all through the parks of Stonia. They are made so that the length of the paths leading up to them is the same as the height of the monument. This is a Size 2 monument:



Once the monuments and paths have been built they are tiled. The monuments *and* paths are both tiled with the same size square tiles.

1. Make a Size 1 monument and its paths. How many tiles would be needed?
2. Make Sizes 2, 3, 4 & 5 and for each one work out the number of tiles needed to tile the monument and its paths.
3. Organise the results you have so far. Can you predict the number of tiles needed for the Size 10 monument? Check by drawing.
4. Imagine the Size 100 monument. How many tiles does it need? Explain your answer.

$$f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+, f(x) = 8x + 1$$



How many tiles for a size 30 monument?

$$f(30) = 30 \times 8 + 1 = 241$$

Continuous v Discrete problems

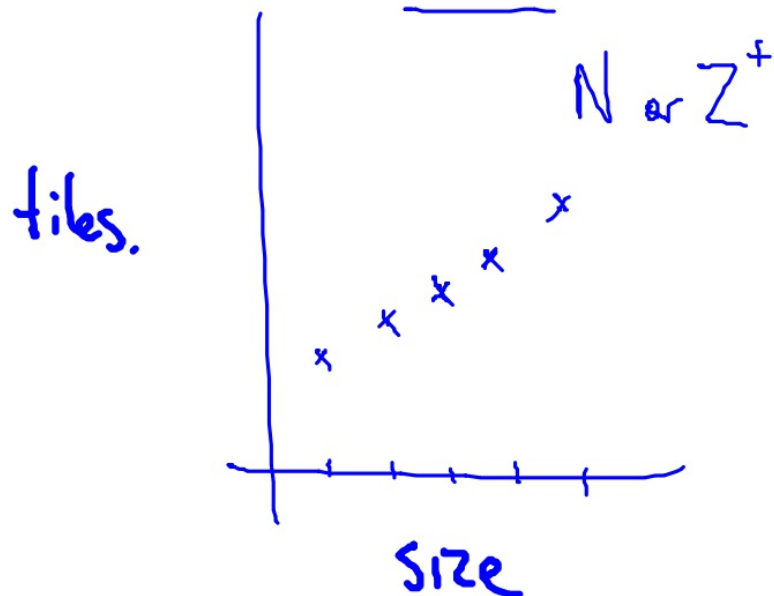
7. What if a monument was built to a height of $5\frac{1}{2}$ blocks, or maybe 6.2 blocks? How could you find the number of tiles needed?

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^+, f(x) = 8x + 1$$

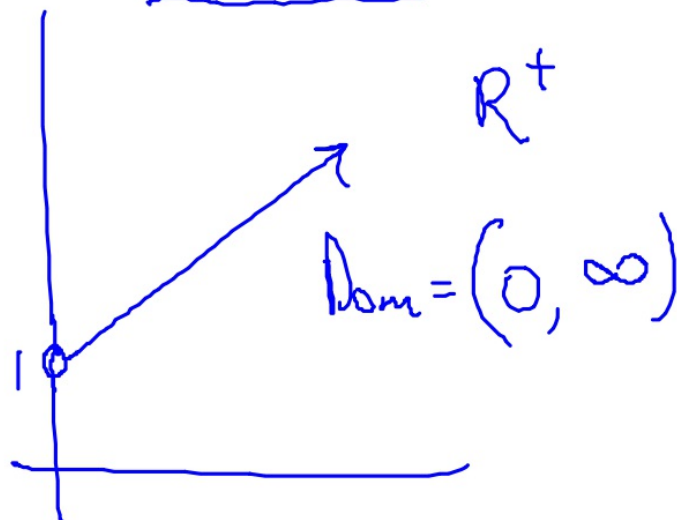
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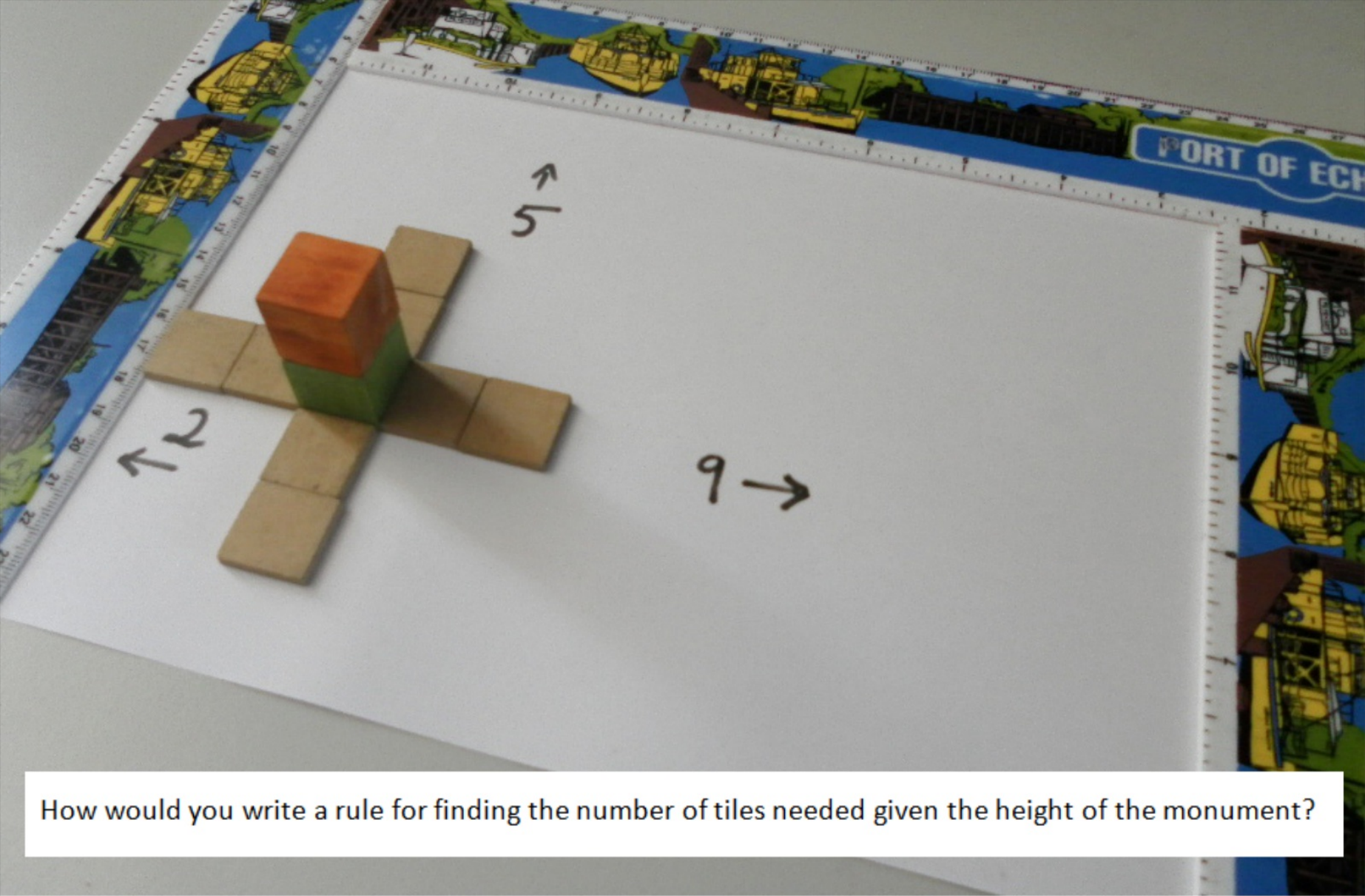
Compare the graphs and the notation of discrete and continuous functions:

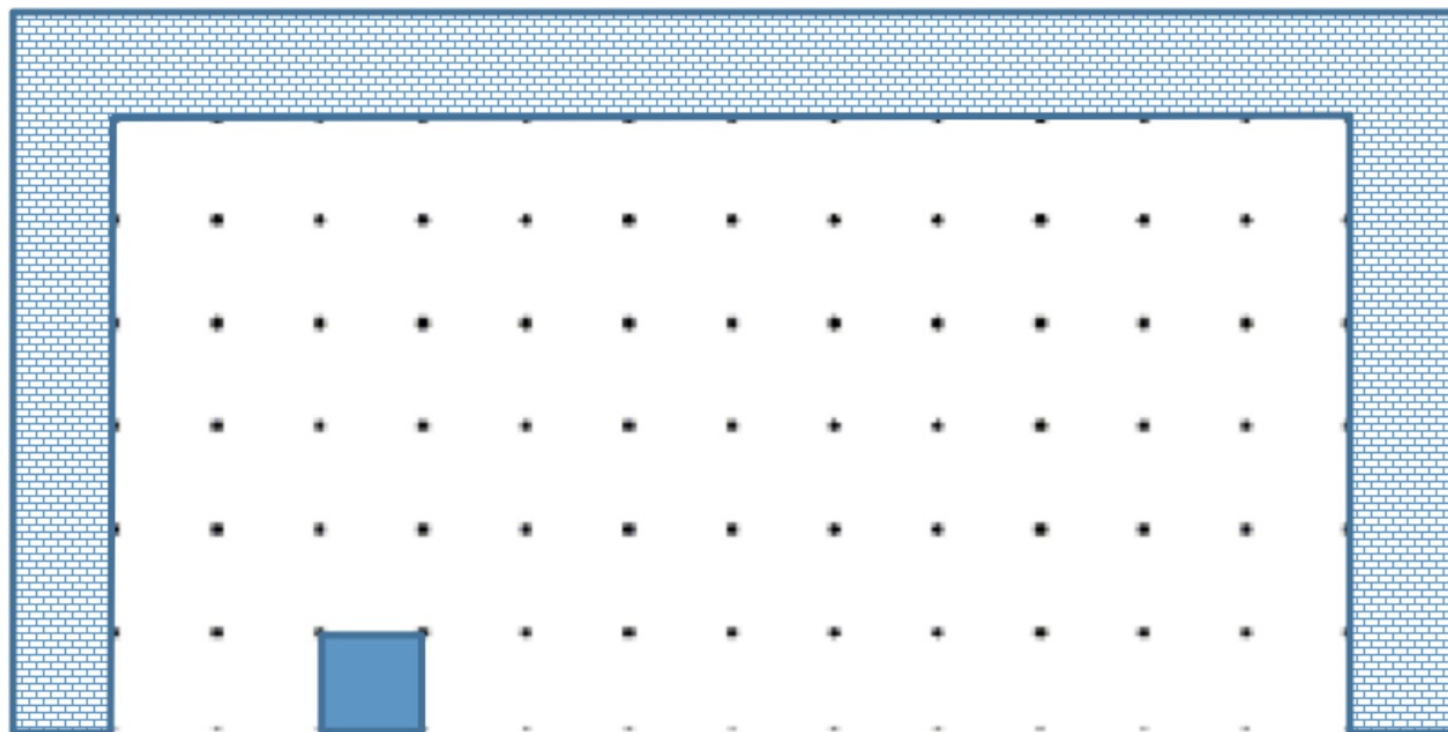
Discrete



Continuous







How would you write a rule for finding the number of tiles needed given the height of the monument?

Make a table:

	1	2	3	4	5	6	7	8	9	10			
	9	17	24	32	40	46	54	62	70	77			

31 38

Break the problem into parts.

① Until we hit the '2' wall. $f(x) = 8x + 1$
 $x \in \{1, 2\}$

② After the '2' wall, until we hit the '5' wall.

$$f(x) = 7x + 3, x \in \{3, 4, 5\}$$

$$f(x) = \begin{cases} 8x+1 & , 0 < x \leq 2 \\ 7x+3 & , 2 < x \leq 5 \\ 6x+8 & , 5 < x \leq 9 \\ 5x+17 & , x > 9 \end{cases}$$

x	5	1	2	3	4	5	6	7	8	9	10	11	12	13
$f(x)$	7	9	17	24	31	38	44	50	56	62	67	72	77	82

The function for tiling our monument in the courtyard would be

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^+, \quad f(x) = \begin{cases} 8x+1 & , 0 < x \leq 2 \\ 7x+3 & , 2 < x \leq 5 \\ 6x+8 & , 5 < x \leq 9 \\ 5x+17 & , x > 10 \end{cases}$$

When we express functions this way they are called **HYBRID** functions (or piecewise functions) .

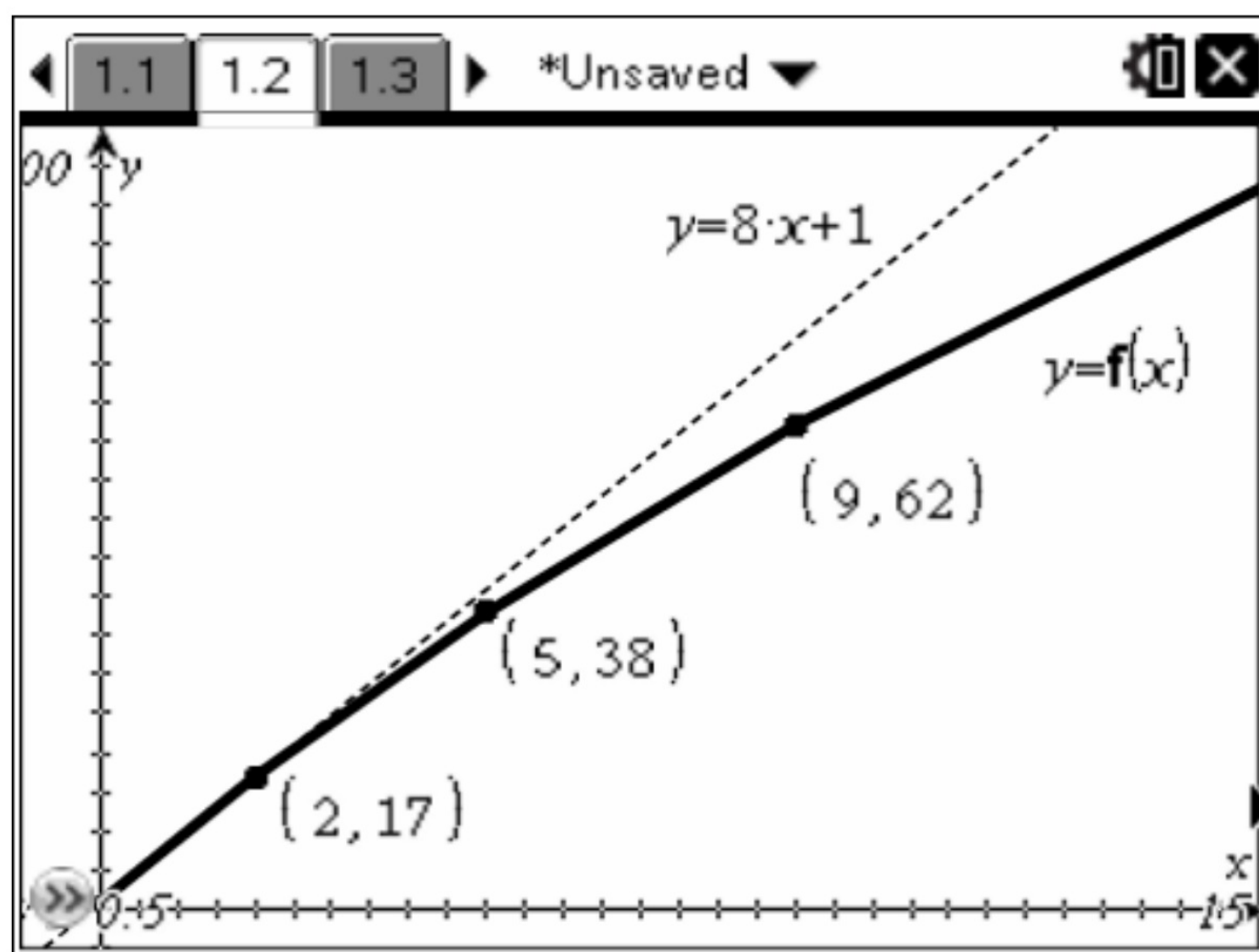
Define $f(x) = \begin{cases} 8 \cdot x + 1, & 0 < x \leq 2 \\ 7 \cdot x + 3, & 2 < x \leq 5 \\ 6 \cdot x + 8, & 5 < x \leq 9 \\ 5 \cdot x + 17, & x > 9 \end{cases}$ Done

$f(6)$	44
--------	----

$f(4)$	31
--------	----

$f(12)$	77
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maths 300

Supporting excellence in mathematics teaching

FARMYARD FRIENDS

You Need

- Five [5] animals - horse, pig, cow, chicken, goat

Animal Pens

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Your Task

1. Arrange the animals in the pens so that:
 - the cow is beside the chicken
 - the horse is in the pen at the end
 - the pig comes after the cow
 - the goat is beside the chicken
2. Find a different solution.

Challenge

How many solutions are there?
How do you know when you have found them all?

© Mathematics Task Centre 2011

Task 129

Arrange the animals in the pens so that

- the cow is beside the chicken
- the horse is in the pen at the end
- the pig comes after the cow
- the goat is beside the chicken

Is there another solution?

How many are there?

How will you know you have found them all?



Farmyard Friends

otherwise entitled
“The Case of the Invisible Chickens”.





the cow is beside the chicken



the horse is in the pen at the end

the pig comes after the cow



the goat is beside the chicken.





PERMUTATIONS.

GoAT	CHICK	Cow	Pig	Horse
Horse	GoAT	CHICK	Cow	Pig
Cow	CHICK	GoAT	Pig	Horse.
Horse	Cow	CHICK	GoAT	Pig
Pig	Cow	CHICK	GoAT	Horse
Pig	GoAT	CHICK	Cow	Horse
Horse	Pig	GoAT	CHICK	Cow.
Horse	Pig	Cow	CHICK	GoAT

Puzzle A

- Suppose only one clue is given:



The cow is in the pen at the end.



GOAT

GOAT

CHICK

CHICK

HORSE

HORSE

CHICK

HORSE

GOAT

HORSE

GOAT

CHICK

HORSE

CHICK

HORSE

GOAT

CHICK

GOAT

$$\begin{array}{r} (24) \\ \times 2 \\ \hline 48 \end{array}$$

(6)



4

3

2

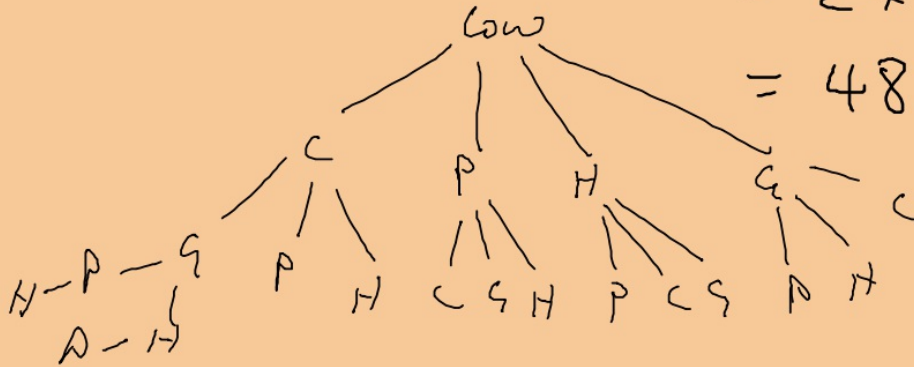
1



$$4 \times 3 \times 2 \times 1 = 4!$$

"4 factorial"

$$\begin{aligned} \text{N}^{\circ} \text{ permutations} &= 2 \times 4! \\ &= 2 \times 24 \\ &= 48. \end{aligned}$$



No restrictions

5 animals.

5 positions.

$$5! = 120$$

Puzzle B

- Another one line clue:



The horse is not next to the pig.



3

2

1

$$12 \times 3!$$



$$= 12 \times 6$$

COMPLEMENT = HORSE NEXT TO PIG

$$= 72$$

$$= 4! \times 2 = 48.$$

$$5! - 2 \times 4! = 72$$

(no restrictions) (complement) (answer we want)

Puzzle C



- Multiple clues:



The cow is in the end pen....

...and the goat is not next to the cow.

$$2 \times 4! = 48.$$

$$2 \times 3! = 12$$

$$\begin{aligned} & 2 \times 4! - 2 \times 3! \\ &= 48 - 12 \\ &= 36. \end{aligned}$$

$$3! \times 3 \times 2$$

C	3	2	1	9
---	---	---	---	---

6

C		9		
C			9	
C				9
		9		C
	9			C
9				C

- 3!

Probability

- If the animals wandered in from the field and randomly selected their pens, what is the *probability* that...
- ...the horse is next to the pig.
- ...the chicken is not in the end pen.

Even more puzzles

- What if the five animals wandered in from the field but there were only three pens?
- *In how many ways could the pens be occupied?*

5 animals, 3 pens

5	4	3
---	---	---

$$5 \times 4 \times 3 = 60$$

n animals, r pens.

$$\frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = \frac{5!}{2!}$$

7 animals, 2 pens.

$$\boxed{7 \mid 6} = 42$$

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5 \times 4 \times 3 \times 2 \times 1}$$

10 animals, 4 pens.

$$\boxed{10 \mid 9 \mid 8 \mid 7} = 5040$$

$$= \frac{7!}{5!} \quad \frac{10!}{6!}$$

If I have n things available, and arrange only r things at a time, the number of permutations is :

$${}^n P_r = \frac{n!}{(n-r)!}$$

Eg.

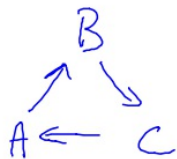
$${}^{10} P_3 = \frac{10!}{(10-3)!}$$

$$= \frac{10!}{7!}$$

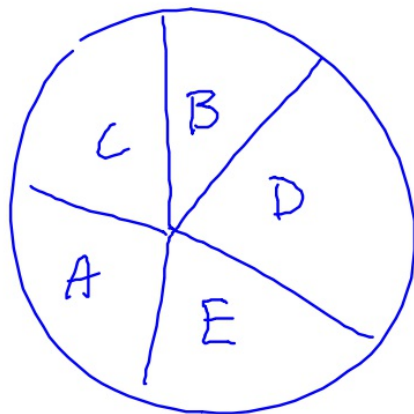
$$= \frac{10 \times 9 \times 8 \times \cancel{7!}}{\cancel{7!}} = 720.$$

What if the pens were arranged in a circle?





A B C
B C A



A C B D E

C B D E A

B D E A C

D E A C B

E A C B D

5 things in a
row would give

$5!$ or 120

$$\frac{120}{5} = 24$$

24 unique circular
arrangements.

COMBINATORICS

6 things in
a circle?

$$\frac{6!}{6} = \frac{\cancel{6} \times 5!}{\cancel{6}} = 5!$$

10 things in
a circle?

$$\frac{10!}{10} = \frac{\cancel{10} \times 9!}{\cancel{10}} = 9!$$

n things in
a circle?

$$(n-1)!$$

What if there were 8 animals and 8 pens, but 3 of the animals were identical chickens?

H G F R	H F G R	G H F R
H G R F	G F H R	G H R F
H R G F	G F R H	
H R F G	G R H F	
H F R G	G R F H	

1	2	↓	↓	3	4	↓	5
		A	B			C	
		B	A			C	
		A	C			B	
		C	B			A	
		C	A			B	
		B	C			A	

3!

$$\frac{8!}{3!}$$

If there are n objects, and
 k objects are identical

the number of unique permutations

is
$$\frac{n!}{k!}$$

n objects

k_1 identicals

k_2 identicals

\vdots

$$\frac{n!}{k_1! k_2! k_3! \dots}$$

① B ~~C~~ D E F

Factorial $n! = n \times (n-1) \times (n-2) \times \dots \times 3 \times 2 \times 1$

In a circle: $(n-1)!$ \swarrow in a row.

Identical objects: $\frac{n!}{k_1! k_2! \dots}$

${}^n P_r = \frac{n!}{(n-r)!}$ (n objects taken r at a time)

Box technique 

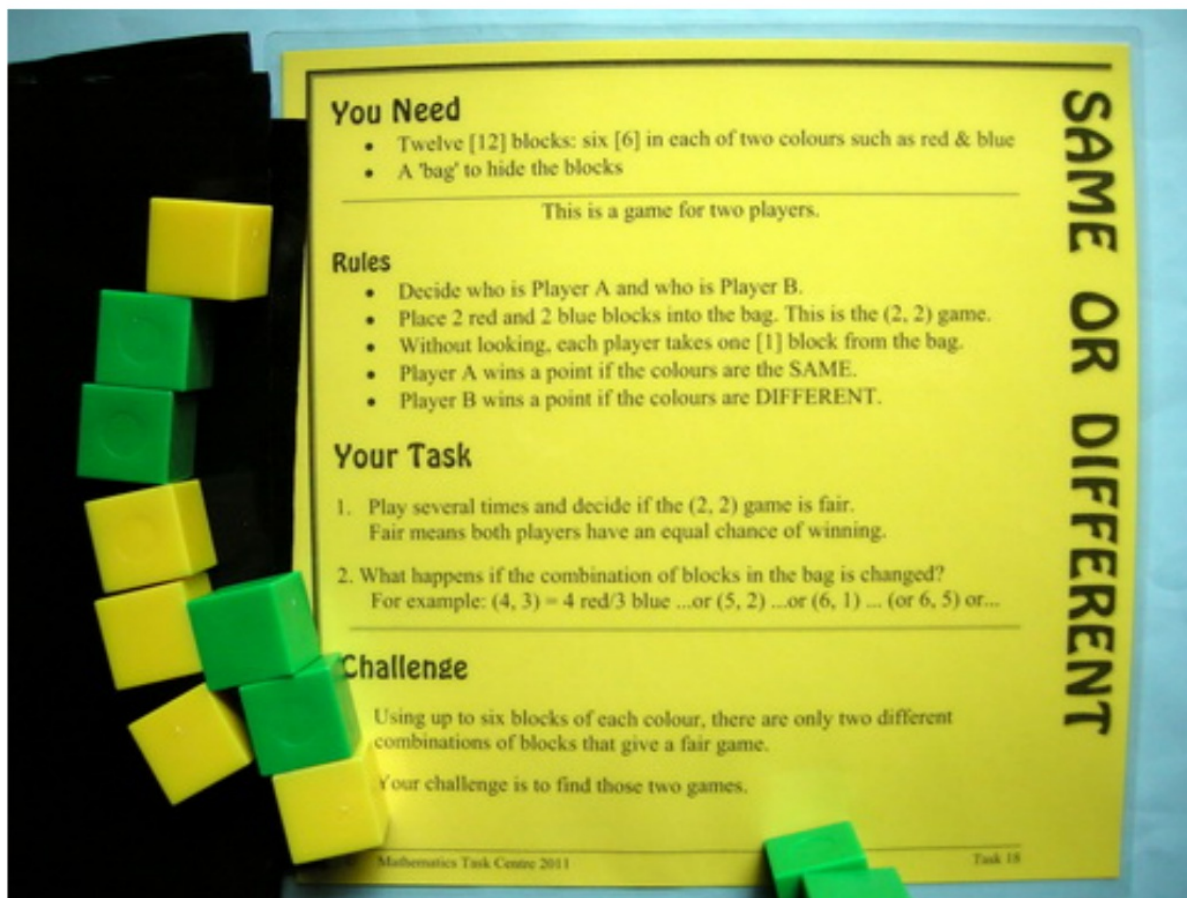
Using the complement. — calculating the opposite.
because it's more efficient.

What if there were 7 pens available,
but only 4 animals to occupy them?

What if there were 7 pens available,
but only 4 animals to occupy them?

$$\frac{7!}{3!} = 840$$

invisible
chickens.



same or different

You Need

- Twelve [12] blocks: six [6] in each of two colours such as red & blue
- A 'bag' to hide the blocks

This is a game for two players.

Rules

- Decide who is Player A and who is Player B.
- Place 2 red and 2 blue blocks into the bag. This is the (2, 2) game.
- Without looking, each player takes one [1] block from the bag.
- Player A wins a point if the colours are the SAME.
- Player B wins a point if the colours are DIFFERENT.

Your Task

1. Play several times and decide if the (2, 2) game is fair.
Fair means both players have an equal chance of winning.
2. What happens if the combination of blocks in the bag is changed?
For example: (4, 3) = 4 red/3 blue ...or (5, 2) ...or (6, 1) ... (or 6, 5) or...

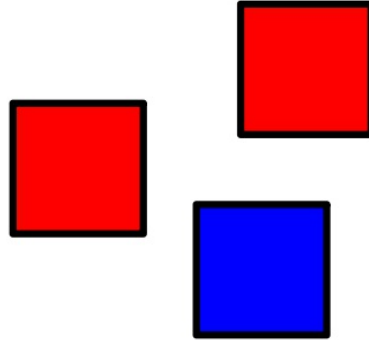
Challenge

Using up to six blocks of each colour, there are only two different combinations of blocks that give a fair game.

Your challenge is to find those two games.

Mathematics Task Centre 2011 Task 18

Draw two blocks out of the bag.
You win if the blocks are the same colour?



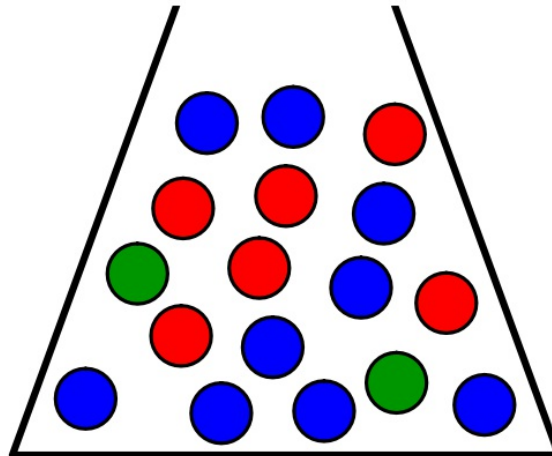
Is this game fair?

If yes, why?

If no, how can
you make it fair?

Same or Different?

Exploration
(Unit 1 / 2 Math Methods)



Introduction to the (1,2) game.

A	B
SAME	DIFF
7, 5, 6	22, 16, 23
10, 12, 8	22, 24, 8
(48)	(115)
(163)	

Empirical data

$$Pr(\epsilon) = \frac{n(\epsilon)}{n(S)}$$

$$\epsilon \equiv \text{SAME}$$

$$\begin{aligned} Pr(\text{SAME}) &= \frac{n(\text{SAME})}{n(\text{TOTAL})} \\ &= \frac{48}{163} \\ &= 0.294 \\ &\text{or } 29.4\% \end{aligned}$$

$$\begin{aligned}
 \Pr(\text{DIFF}) &= 1 - \Pr(\text{SAME}) && (\text{COMPLEMENT}) \\
 &= 1 - 0.294 \\
 &= 0.706
 \end{aligned}$$

(2,2) game

SAME	DIFF
11, 13, 15	19, 21, 20
15, 15,	25, 33,

Setting the challenge. Find fair games empirically.

Game	$Pr(S)$	$Pr(D)$
(2, 1)	0.294	0.706
(2, 2)	0.369	0.631

CHALLENGE:

SEARCH FOR A
FAIR GAME.

Combinations found empirically that we could

- ☐ a) support with the computer software,
- ✓ b) confirm using probability theory

COMBINATIONS

WE WANT TO TEST

FOR FAIRNESS.

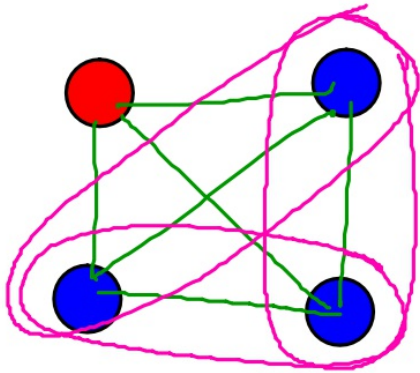
$(2, 3)$ $(3, 3)$ $(2, 4)$ $(6, 4)$

$(3, 4)$ $(3, 5)$ $(3, 6)$ $(10, 5)$

$(4, 8)$ $(1, 3)$ $(12, 8)$ $(9, 13)$

✓

Using probability theory to prove fairness



6 possible pairs.

3 pairs are same.

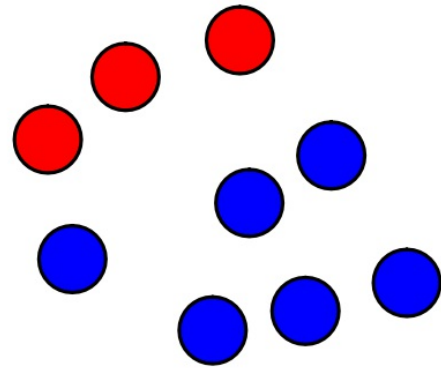
$$\begin{aligned} \Pr(\text{same}) &= \frac{n(\text{same})}{n(\text{total})} \\ &= \frac{3}{6} = \frac{1}{2} \end{aligned}$$

Same 3C_2

total 4C_2

$$\begin{aligned} \Pr(\text{same}) &= \frac{{}^3C_2}{{}^4C_2} \\ &= \frac{3}{6} = \frac{1}{2} \end{aligned}$$

$(3, 6)?$



Same ${}^6C_2 + {}^3C_2$
total. 9C_2

$$\begin{aligned} P_r(\text{same}) &= \frac{{}^3C_2 + {}^6C_2}{{}^9C_2} \\ &= \frac{1}{2} \end{aligned}$$

Games that appeared fair on the computer were proved to be close, *but not perfect.*

$(8, 12)$

$$\begin{aligned} \Pr(\text{same}) &= \frac{\binom{8}{2} + \binom{12}{2}}{\binom{20}{2}} \\ &= \frac{47}{95} \quad \text{close!} \end{aligned}$$

$(9, 13)$

$$\Pr(\text{same}) = \frac{38}{77} \quad \text{close!}$$

$$(6, 9) \rightarrow \frac{17}{35} \quad \text{close!}$$

These combinations make perfectly fair games:

$(1, 3)$ $(3, 6)$ $(6, 10)$ $(10, 15)$

Pairs of triangular numbers provide perfectly fair games.

1, 3, 6, 10, 15, 21, 28, 36, ...

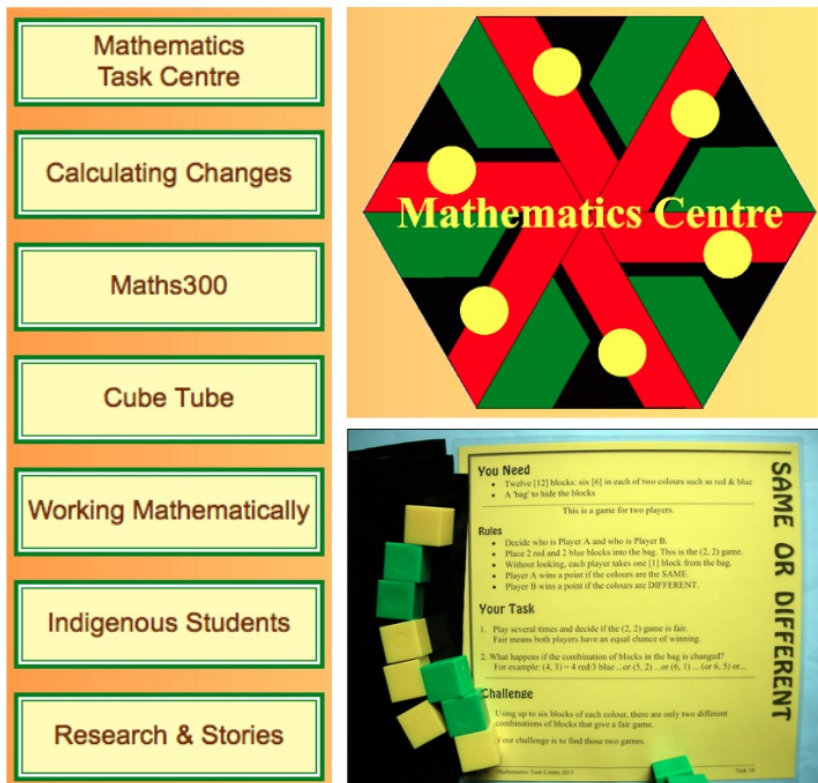
$$\Pr(\text{same}) = \frac{{}^{28}C_2 + {}^{36}C_2}{{}^{64}C_2} = \frac{1}{2}$$

Can you have a fair game with three colours?

Same rules: If I draw two blocks and they are the same colour, I win.

	A	B	C	D	E	F	G	H
1		Number of Blocks	Pairs					
2	BLUE	4	6	=IF(B2>1,COMBIN(B2,2),0)				
3	RED	5	10					
4	GREEN	10	45					
5								
6								
7								
8			=SUM(C2:C4)					
9	Numerator	61						
10	Denominator	171	=COMBIN(SUM(B2:B4),2)					
11								
12	Probability	0.3567						
13			=B9/B10					
14								
15								
16								
17								
18								

<http://mathematicscentre.com/taskcentre/018same.htm>



From The Classroom

Damian has also used *Same Or Different* with his Year 12 class. He was pretty impressed that they suggested using a tool developed for a totally different investigation a year earlier in Year 11. That story is below. This story is about mathematicians asking the question *What happens if...?* then taking an adventure together when even the teacher doesn't know the end point.